Dislocation–impenetrable precipitate interaction: a three-dimensional discrete dislocation dynamics analysis

C. S. Shin††, M. C. Fivel†¶, M. Verdier§ and K. H. Oh‡
† Génie Physique et Mécanique des Matériaux, Institut National Polytechnique de Grenoble, CNRS, BP 46, 38402 Grenoble, France
‡ School of Materials Science and Engineering, Seoul National University, 151-742 Seoul, South Korea
§ Laboratoire de Thermodynamique et Physico-chemie Metallurgiques, Institut National Polytechnique de Grenoble, CNRS, BP 75, 38402 Grenoble, France

[Received 31 July 2002 and accepted 19 February 2003]

Abstract

In an attempt to better understand the effect of the difference between the shear moduli of the particle and matrix on the flow stress and the work hardening, a numerical approach based on discrete dislocation simulations is developed in which the image stress caused by a second phase impenetrable particle on dislocations is implemented. Glide of a dislocation line of initially screw type through a channel between two spherical particles of shear modulus \( G_p \) is simulated. Shear stress is applied incrementally on the slip plane and the equilibrium position of the dislocation line is calculated for the given applied stress. It is found that the flow stress at which the dislocation bypasses the obstacles by bowing between a pair of particles varies as \( (\Delta G/G_m)^{\alpha} \), where \( G_m \) is the shear modulus of the matrix and \( \Delta G \) is the difference between shear moduli. \( \alpha \) is found to be less than 1 and the effect of \( \Delta G \) is amplified as the radius of the spherical particles increases. The stress increment required to force a dislocation to glide between the particles which have remaining Orowan loops from previous slip becomes higher as the particle gets harder. A relationship giving the hardening stress as a function of the number of loops is proposed. Finally, it is found that dislocations can bypass particles by cross-slip as soon as a certain number of Orowan loops surrounding the particles is reached. The image stress field around the particle induced by a difference between the shear moduli seems to enhance the cross-slip probability.

§ 1. Introduction

The hardening of materials caused by distributing small particles of another phase is a well-known phenomenon and has been used to develop high-strength structural materials. The impediment of dislocation glide by second phase-particles is the basic mechanism for increasing the flow stress. In the case of impenetrable particles, the back stress due to trapped closed loops causes the subsequent...
increment of stress, that is work hardening. In addition to the glissile closed loops left around the particles, prismatic loops have also been observed experimentally to form near a particle by cross-slip of dislocations (Humphreys and Martin 1967). Particles in two-phase materials can be classified by size, shape, volume fraction, spatial distribution (regular or random) and characteristic of the particle–matrix interfaces (coherent or incoherent). In addition to these morphological parameters of particles, important parameters to describe the mechanical properties of two-phase alloys are the stress field around a particle, the stress incompatibility due to the difference between the shear moduli and the image stress caused by the change in the strain energy of a dislocation near a second-phase particle.

A number of experimental and analytical research studies have been focused on finding relevant parameters and the effect of those in determining the flow stress and the work-hardening properties of these alloys. For example, well-designed experiments, in which the volume fraction, size and spacing of particles can be varied, have been made to measure the effect of such parameters on the mechanical behaviour of single crystals containing hard particles (Ebeling and Ashby 1966, Humphreys and Martin 1967). Theoretical approaches to account for the effect of relevant parameters have been developed from the Orowan mechanism, which considers the elementary dislocation–particle interaction and relates the flow stress to the obstacle spacing, to the methods for dealing with the anisotropy of material and the complicated statistics such as random distribution of particles. An overview of the analytical approaches can be found in a review article by Reppich (1993). To account fully for the realistic interaction of dislocations and many particles, computer simulations have been developed. Foreman and Makin (1966) have investigated the effect of a random arrangement of strong and weak point obstacles on the flow stress. In recent simulations, more complex and accurate states are treated, for example the distribution of finite particles including the shape effect (Zhu and Starke 1999), and particles with the mismatch stress field caused by a difference between the lattice constants (Mohles and Nembach 2001). More insights on the effect of each parameter on the flow stress can be gained through these simulations. Note that, in the simulations mentioned above, it was generally assumed that dislocations move on a single glide plane so that no three-dimensional (3D) events such as cross-slip were allowed.

It is well known that a force is acting on a dislocation near an interface between two materials of different shear moduli, but it is not a simple task to implement such a force in the computer simulations since it is generated by dynamic interaction between dislocations and particles and there is no simple analytical solution for the image stresses caused by a second-phase particle. In this work, the effect of the image stress on the flow stress and the work hardening has been studied using a 3D dislocation dynamics simulation coupled with a finite-element code. The simulation method is detailed in §2. Glide of a dislocation line through a channel between two incoherent impenetrable spherical particles is considered. Dynamic interaction of a dislocation line with particles and the resulting image stress are solved in the 3D space. Several dislocation lines are forced to move between two spherical particles one at a time and the resolved shear stress needed to bypass the particles having trapped loops is monitored while cross-slip of dislocations is allowed. By changing the shear modulus of the particles keeping parameters such as the particle radius and interparticle spacing constant, the effect of the image stress induced by second phase particles is evaluated and the relation of the flow stress with the difference between the shear moduli is established.
§2. SIMULATION METHOD

In the 3D dislocation dynamics simulation code, a dislocation line is represented as a connected set of discrete dislocation segments of edge and screw type. The dislocation segments move on a simulation network similar to the fcc crystal but different in unit length. The motion of each segment is governed by a resolved shear stress acting on each segment. In the case of a finite-volume simulation, the stress components, which are responsible for the resolved shear stress of the segments, include the following:

(i) the internal stress field $\sigma_{\text{int}}$ produced by all the other dislocation segments present in the simulation box;
(ii) the applied stress field $\sigma_{\text{app}}$, which compensates for the proper boundary conditions on the surface of the finite volume;
(iii) the Peierls forces $\tau_{\text{Peierls}}$, which represent the lattice friction on the dislocation motion;
(iv) the line tension stresses $\tau_l$ of dislocation lines reflecting their self energy.

The effective resolved shear stress $\tau$ acting on each dislocation segment is then computed at the middle point of the segment using the Peach–Koehler equation in the frame of isotropic elasticity. The velocities of each segment follow a Newtonian dynamic $v = \tau b / B$ ($B$ is the phonon drag coefficient) and the new positions of the segments are calculated by integrating the velocity numerically. The local interactions of the dislocation segments, such as junction formation, annihilation and cross-slip are implemented as local rules modelling. Cross-slip of a screw segment is considered in a stochastic manner. Following the Bonneville et al. (1988) considerations, a cross-slip probability $P$ over each time step is computed first using the expression

$$P = \beta \frac{l}{L_0} \frac{\delta t}{t_0} \exp\left(\frac{\tau_d - \tau_{\text{III}}}{\kappa T} V\right),$$

where $\beta$ is a normalization coefficient to make $P$ be between 0 and 1, $l$ is the length of the particular screw segment, $L_0 = 1 \mu m$, $\delta t$ is the discrete time step, $t_0 = 1$ s, $V$ is the activation volume, $\tau_d$ is the resolved shear stress in the deviate glide system and $\tau_{\text{III}}$ is the threshold stress. In the present paper, the material for the matrix containing the dislocations is chosen to be copper. Thus, the parameters used for the dislocation simulations are those commonly used for copper: $b = 2.56$ A, $B = 1.5 \times 10^{-5}$ Pa s, $V = 350b^3$, $\tau_{\text{Peierls}} = 3 \times 10^{-5} G_m$ and $\tau_{\text{III}} = 32$ MPa. In practice, a random number $r$ is generated and the dislocation cross-slip occurs only if $r$ is lower than $P$. Complete details of the 3D dislocation dynamics code used in this paper can be found in the paper by Verdier et al. (1998).

2.1. Calculation of the image stresses

The calculation of the image stresses on a dislocation in the presence of a second-phase particle involves decomposition of the problem into the problem of dislocations in an infinite elastic isotropic medium and the complementary problem of a finite dislocation-free volume. The complementary problem compensates for the proper boundary conditions. In the case of free surfaces, for example, the forces exerted on the surfaces by dislocations are computed assuming that the dislocations exist in an infinite medium. These forces are then reversed and changed into the appropriate point forces to enforce the surface traction free. Applications of this
method can be found in previous work (Fivel et al. 1999). In the current study, the complementary problem of a finite dislocation-free volume is more complex since it consists of matrix and particles. The principle of the decomposition is shown schematically in figure 1. The stress field $\sigma$ at a given point of the volume is the sum of two stress field $\sigma_1$ and $\sigma_2$ solutions of two elastic problems. In the first problem of the decomposition, it is assumed that the dislocations are in an infinite elastic isotropic medium. The stress $\sigma_1$–strain $\epsilon_1$ relationship is expressed as $\sigma_1 = L_M : \epsilon_1$ in the whole volume, where $L_M$ represents the fourth-rank stiffness tensor of the matrix and the dislocation stress field $\sigma_1$ is given by the DeWit (1967) formula. The relation is incorrect in the volume $(V_p)$ representing particles. So the complementary problem should have a correction term such as $\sigma_{\text{correc}} = (L_p - L_M) : \epsilon_1$ in $V_p$ to compensate for the wrong stress field in the particle volume of the first problem. Replacing $\epsilon_1$ by $L_M^{-1} : \sigma_1$, this correction term equals to $(L_p : L_M^{-1} - I) : \sigma_1$, where $L_p$ and $I$ represent the fourth-order stiffness tensor of the particle and the fourth-order unit tensor respectively. The governing equations for the complementary problem become

$$\sigma_2 = L_M : \epsilon_2 \quad \text{in} \quad V_M,$$
$$\sigma_2 = L_p : \epsilon_2 + \sigma_{\text{correc}} \quad \text{in} \quad V_p. \tag{2a}$$

$$= L_p : \epsilon_2 + (L_p : L_M^{-1} - I) : \sigma_1 \quad \text{in} \quad V_p. \tag{2b}$$

Previous applications of this method to two-dimensional (2D) and 3D cases can be found in the papers by Cleveringa et al. (1997) and Shin et al. (2001). The complementary problem is solved using CAST3M, a finite-element code developed in France by the Commissariat à l’Énergie Atomique. The stresses by dislocations at the points in $V_p$, for example at the stress integration Gauss points, are calculated and the correction term $(L_p : L_M^{-1} - I) : \sigma_1$ is computed within each finite element. The calculated stresses are then changed into a nodal body force field $f^b$. These forces are applied to $V_p$, and the finite-element method (FEM) gives the solution of a two-phase boundary problem where forces and displacements are imposed at the

Figure 1. Decomposition of the problem into the problem of dislocations in an infinite media and the complementary problem of an inhomogeneous finite volume without dislocations. Forces $F_an$ and displacements $U_an$ are applied on the boundary. In the complementary problem, the boundary conditions are modified with the forces $F_D$, the displacements $U_D$ and the nodal body force field $f^b$ generated by the dislocations.
boundary and body forces are applied inside the particle. The continuity of displacements and normal stresses at the particle–matrix interface is enforced by the FEM.

2.2. Calculation procedures

The problem considered here consists of two impenetrable rigid spherical particles having radius $R_p$, interparticle distance $L$ and shear modulus $G_p$. A typical 3D simulation box is shown in figure 2. A single-slip system and the parameters are also indicated in the figure. In the dislocation code, the spherical particles are modelled as a set of facets made of polyhedral surface elements. Each facet of the spherical particle has a certain strength to act as an obstacle to dislocation motion; that is, a dislocation is authorized to cross the facet if the local effective resolved shear stress is above a critical strength. In this study, the strength of facets has been chosen sufficiently high that it represents impenetrable hard obstacles. That way, it is possible to use a simplified version of the code for which no image stress is computed. The interaction between dislocations and particles is then only related to an obstacle effect. This situation is later referred to as the $\Delta G = 0$ case. In order to include the effect of the image stress, one has to solve the complementary problem described in figure 1. To do so, a 3D box containing two spherical particles has been meshed in the code CAST3M. To represent correctly the high gradient of stress near the particles, the mesh has been refined near the periphery of the particles. A typical section of the 3D mesh at the centres of particles is shown in figure 3. The displacement of the bottom surface is set to be zero in normal direction of the surface and two nodes located on this surface are frozen in adapted directions in order to remove the trivial rigid-body solution. A dislocation line, which is initially a pure screw segment, is fixed at two end points. The position of the pinned points is set to be 0.9 of $L/2$ from the border of the particle so that the portion of the dislocation line which lies between the particles bypasses the particles by bowing out. In § 3.1, it will be shown that this fixed point can be used to obtain reliable results for the flow stress. A single slip on the (111)[1–10] system is assumed. The resolved shear stress $\tau$ on the

Figure 2. A simulation box used in the 3D discrete dislocation simulation. Two particles of radius $R_p$ and interparticle distance $L$ are shown with a dislocation line on the (111)[110] slip system.
slip plane is raised in small steps in a quasistatic manner. After each increment, the positions of all segments are computed as a function of time until the shear strain $\gamma$ caused by the dislocation motion has fallen below a certain value. If the dislocation line has reached an equilibrium position, $\Delta \gamma$ is nearly equal to zero, $\tau$ is then increased and $\Delta \gamma$ is monitored again. After the dislocation line has bypassed completely the particles leaving two Orowan loops, a new dislocation line is introduced and the subsequent increment of $\tau$, that is the work hardening, is computed.

§ 3. Results

In order to check the accuracy of the solution given by the finite-element calculation, the stress exerted on a long straight edge dislocation near a circular hole has been calculated and compared with the work of Chen et al. (1999). These workers have formulated the image stress of this problem in the 2D condition of plane strain and with a straight infinite edge dislocation in an isotropic body. In order to reproduce this situation, the copper matrix ($G_m = 42$ GPa; $\nu = 0.324$) containing a cylindrical hole of 0.262 $\mu$m radius has been meshed using 20-node 3D elements, and a long straight edge dislocation has been put near the hole. The stress exerted on the dislocation line has been calculated with the FEM mentioned above. Note that this problem could be solved using the point forces compensation method as proposed by Fivel et al. (1996), but the idea here is to check the finite-element procedure based on the body force compensation as detailed in § 2.1. Thus the hole is represented here by imposing a shear modulus equal to $10^{-4}$ that of the matrix. Several meshes have been tested. Good results are obtained for the mesh consisting of 1150 20-node
elements, the size of the smallest element being 20 nm. In figure 4, the calculated image stress acting on the dislocation line is compared with the analytical solution.

Using the same procedure, a spherical hole of the same radius has been meshed and the image stress caused by the spherical hole has been calculated. As shown in figure 4, it is found that the image stress exerted by a spherical hole is lower than the stress caused by a cylindrical hole and decreases more rapidly; for example, the stress exerted on the dislocation at 0.2882 μm from the spherical hole centre is three quarters of the stress near the cylindrical hole and one quarter when the dislocation is located at 0.4978 μm. The mesh used in this calculation has been applied to all the following simulations.

3.1. Flow stress of impenetrable particles with different shear moduli

In order to validate the code used in this work, the flow stress of impenetrable particles with no image stress has been calculated and compared with the results of Bacon et al. (1973). As explained in § 2.2, no finite-element procedure is used here. The particles are modelled by the facet obstacles. The radius of particles are set to 0.131, 0.262 and 0.524 μm with a fixed interparticle distance $L = 2.59 \mu m$. The situation considered here is similar to the models used by Bacon et al. (1973) except for the periodic boundary condition. Figure 5 shows our results of the dislocation configurations at the flow stress for three particle radii respectively. The dislocation line near the particles is in a parallel position owing to the self-interaction which pulls the branches on opposite sides of the particle and the line between the particles is quite symmetric. The equilibrium line shapes depend strongly on the radius of the particle. In figure 5, the maximum bow-out length is measured in units of $L$ and the

![Figure 4](image.png)

Figure 4. Image stress (in units of $K \left( \frac{G_m b}{4\pi(1 - \nu)R_p} \right)$) exerted on a long straight edge dislocation near a cylindrical (×) and a spherical hole (+). The dotted curve represents the analytical solution given by Chen et al. (1999). Both the cylindrical and the spherical hole have radii $R_p$ of 0.262 μm.
values are 0.482, 0.554 and 0.607 for particle radii of 0.131, 0.262 and 0.524 \mu m respectively. The increase in maximum bow-out length with the particle radius corresponds well to the results of Bacon et al. (1973). The results obtained for the flow stress have been compared according to the ‘effective line tension’ argument proposed by these workers. They argued that the effective line tension, which properly accounts for the self-interactions, can be taken as 

\[ A \left[ \ln \left( \frac{b + b}{L} \right)^{-1} + B \right], \]

where A is \( \frac{1}{2\pi} \) and \( \frac{1}{2\pi}(1 - \nu) \) for an edge dislocation and a screw dislocation respectively. Figure 6 shows the results obtained for the flow stress normalized by \( G_m b/L \) plotted against \( \ln \left( \frac{b + b}{L} \right)^{-1} \). The linear relation is perfectly reproduced and the slope of the fitting line is about 0.254, which is close to the expected value of \( \frac{1}{2\pi}(1 - \nu) \). Considering these observations, it can be said that the fixed dislocation source at the point 0.9 of \( L/2 \) from the periphery of the spherical particle represents well the periodic boundary condition used by Bacon et al. (1973).

To investigate the effect of the difference between the shear moduli on the flow stress, we have made simulations of an alloy of the copper matrix containing two spherical particles of radius \( R_p = 0.262 \mu m \). A shear modulus ratio \( (\Delta G/G_m) \) was set to 1, 3 and 5, where \( \Delta G = G_p - G_m \). The interparticle distance is fixed to \( L = 2.59 \mu m \). Figure 7 shows the increment in the flow stress as a function of \( \Delta G/G_m \). As the shear modulus of the particles increases, the flow stress increases because the repulsive image stress on the dislocation line needs a higher resolved shear stress to bypass the dislocation line through the particles. The fitting curve shows that the flow stress changes as \( (\Delta G/G_m)^{0.6} \). The change in the flow stress is small even for the particles with a shear modulus 6\( G_m \), for which the shear stress only increases by about 6% compared with the no-image-stress case. In fact, a minute effect of \( \Delta G \) is expected from the short range of the image stress. Indeed, calculations show that the image
Figure 6. $\tau_{ys}/(G_m b/L)$ versus $\ln(b/(2R_p + b/L))^{-1}$. The line represents the fitting line and the slope is 0.254.

Figure 7. Increase in the flow stress due to the difference between the shear moduli. $\Delta \tau/\tau_0$ is plotted against $\Delta G/G_m$, where $\tau_0$ represents the flow stress of the impenetrable particles with no image stress ($R_p = 0.262 \mu m$). The fitting curve shows that $\Delta \tau \propto (\Delta G/G_m)^{0.6}$. 
stress exerted on a long straight dislocation line decreases as $|x - x_0|^\alpha$, where $x$ and $x_0$ represent the position of the dislocation line and the centre of a spherical particle respectively. $\alpha$ is found to be around 6–7. Thus, even in the case when $\Delta G/G_m = 5$, the image stress decreases below the flow stress of hard obstacle ($\Delta G = 0$) at a distance of $1.4R_p$ from the centre of the particle, which means that the repulsive interaction between the dislocation and the particle will reduce the interparticle spacing by about 8% in this case. The small reduction in the interparticle spacing will result in the small increase in the flow stress.

The effect of a difference in shear modulus on the increment of the flow stress can be summarized as $\tau_{ys} \propto (\Delta G/G_m)^\alpha$, where $\alpha$ is lower than 1. Note that this result is different from the case of shearable particles. Indeed, the effect of shear modulus of coherent shearable particles can be found in the literature (Nembach 1983). He has calculated approximately the image stress on a straight infinite dislocation line due to spherical particle using the change in the strain energy density of a dislocation caused by a particle of modulus $G_p$. The shear stress is found to be proportional to $\Delta G^{1.5}$ and $R_p^{0.22}$. Thus, we observed that $\Delta G$ has weaker effect in the case of hard particles than in the case of shearable particles.

3.2. Increment in hardening stress

Although the difference in shear moduli has little effect on the flow stress, it affects greatly the hardening stress. In this section, the effect of $\Delta G$ on the hardening stress is discussed. The stress required to force a dislocation to glide between the particles, having the remaining Orowan loops, is plotted in figure 8 for the case when $G_p = 4G_m$ (full symbols) and no image stress ($\Delta G = 0$ (open symbols)). The change in the shear modulus of the particles results in an increase in the work-hardening rate. The effect of $\Delta G$ is enhanced as the radius of the particle becomes larger. The image stress field is the sum of the interactions of the particles and each dislocation loop existing around the particles. So the dislocation line bypasses the particles in the additional image stress field coming from the interaction of the residual loops and the particles. This additional stress is directly related to the number of Orowan loops stored around the particles. As a result, compared with the no-image-stress case, a higher shear stress is required to bypass the particles and this effect is more pronounced as more dislocation lines are passing, which leads to a more enhanced hardening of the material. Considering that the range and the magnitude of the image stress increase as the radius $R_p$ increases, it can be understood that the hardening rate is increased as $R_p$. Fisher et al. (1953) have investigated the hardening of metal crystals by precipitate particles. They computed the back stress resulting from the Orowan loops and calculated the effective critical stresses of Frank–Read sources. The argument is that the hardening stress $\tau_h$ is related to the number ($N$) of loops by $\tau_h = \gamma N$, where $\gamma$ is a function of particle radius $R_p$ and interparticle distance $L$:

$$\gamma = c b G_m \left(1 - \frac{v}{2(1 - v)}\right) \frac{R_p^2}{(L + 2R_p)^3}.$$  

(3)

$c$ is the parameter describing the closest distance of a source and a particle. We obtained the slopes of each graph in figure 8 by linear fitting and the dependence of these slopes on $b(R_p)^2/(L + 2R_p)^3$ is given in figure 9 showing the linear dependence. Thus, it is found that the argument of back stress by Fisher et al. (1953)
Figure 8. Work hardening of the alloy containing two particles of radii 0.131 µm (△), $\Delta G/G_m = 0$; (▲), $\Delta G/G_m = 3$; 0.262 µm (□), $\Delta G/G_m = 0$; (■), $\Delta G/G_m = 3$ and 0.524 µm (◇), $\Delta G/G_m = 0$; (◆), $\Delta G/G_m = 3$. $\tau_h - \tau_f$ is plotted against the number of Orowan loops for each particle, where $\tau_h$ and $\tau_f$ represent the hardening stress and the flow stress respectively.

Figure 9. The slope of the fitting line in figure 8 versus $bR_p^2/(L+2R_p)^3$: (∗), $\Delta G = 0$; (×), $\Delta G = 3G_m$. 
still holds in the case of a moving dislocation line through two particles. The parameter \( c \) is around 2.24 for the case of no image stress and 2.88 when \( G_p = 4G_m \), which means that, based on their argument, the effective distance of a stress source is shorter or the back stress is higher if the image stress is included.

It is observed experimentally that dislocations left around particles by gliding dislocations are not rigorously confined to a single glide plane but can relax the elastic energy by adopting prismatic configurations (Humphreys and Martin 1967, Kelly and Nicholson 1971). If cross-slip is easy, a dislocation may avoid an obstacle in its glide plane by slipping on another slip plane with the formation of long jogs. The simulations presented above to investigate the change in hardening stress have been made under the condition that cross-slip of the dislocation is prohibited by artificially changing the parameters for cross-slip. When the normal conditions for cross-slip are used, cross-slip events have been observed. As an example for the particles of the radius 0.262 \( \mu m \), cross-slip occurs if the number of Orowan loops reaches four in the case of no image stress and two in the case of \( G_p = 4G_m \). Considering that the back stress on the primary slip plane becomes higher as the accumulation of the Orowan loops proceeds, it is easy to move on to the secondary slip plane and to resume its motion on another plane of the original slip system (double cross-slip) to bypass the particle. Figure 10 shows the bypassing of the dislocation line by double cross-slip. If the shear modulus of the particle is higher than the matrix, a high local stress is generated near the particle and the local event of cross-slip is more probable by the effect of the image stress. This demonstrates that it is important to include the image stress to investigate local events such as cross-slip.

§4. Discussion

In this work, we studied the effect of the difference between the shear moduli on the flow stress and the subsequent hardening stress using the 3D discrete dislocation dynamics code. Firstly, the method of image stress calculation has been verified by

![Figure 10. Bypassing the particles by double cross-slip of a dislocation line. The primary \((111)[110]\) and the secondary \((111)[110]\) slip systems are indicated.](image)
comparison with the analytical solution of a straight infinite edge dislocation near a circular hole (Chen et al. 1999). The solution obtained using the decomposition principle represents well the analytical solution when meshes based on 20-node elements are used. The image stress caused by a spherical hole has also been studied and is found to decrease more rapidly than in the case of a cylindrical hole. The effect of $\Delta G$ on the flow stress has been investigated using the same method. This effect can be summarized by $\tau_y \propto (\Delta G/G_m)^\alpha$, where $\alpha$ is lower than 1. Because the range of the image stress is short, the maximum increment in the flow stress is only 6% in the case when $\Delta G/G_m = 5$ when compared with the no-image-stress case, but the effect of the image stress increases as the Orowan loops are accumulated to change the work-hardening rate. This result arises because the image stress field is the sum of interactions of the particle and of each dislocation loop existing around the particle. So, as slip accumulates, a dislocation line feels an additional image stress field coming from the interaction of the residual loops and the particles. The first-order approximation for the work hardening given by Fisher et al. (1953) is still found to be valid in the simple configuration of two particles and a dislocation line. The effect of the image stress is that the effective distance of a stress source becomes shorter or the back stress becomes larger. If cross-slip of a dislocation is allowed in the code, it has been observed that the dislocation can avoid the obstacles in its glide plane by moving on to another slip plane. The back stresses on the original glide plane caused by the Orowan loops cause cross-slip. If the image stress is included, it is more probable that cross-slip will happen. So the image stress around the particle is high enough to affect local events such as cross-slip.

The requirements for computation time and effort are too demanding to include the effect of an image stress on simulations of an alloy containing a large number of particles. The number of elements used here in the simple situation of two particles is already about 5000 20-node elements. An approximate way to include the effect of the image stress is to introduce an effective radius which can represent the difference between the shear moduli. That way, the facet obstacles alone can be used to reproduce the precipitate hardening; that is, no finite-element coupling is needed. For example, in the case when $\Delta G = 3G_m$, the average radius of the first trapped loop is 0.272 $\mu$m compared with 0.264 $\mu$m for the case when $\Delta G = 0$. The difference between the shear moduli changes the effective radius by a few per cent, but the stress field generated by a second phase particle increases as the slip accumulates and the image stress field turns out to be crucial to predict the property of work hardening. This problem can then be treated using an empirical solution of the image stress generated by the interaction between several dislocations and a particle.

References