FORMING LIMIT DIAGRAM OF PERFORATED SHEET

Seung Chul Baik*, Kyu Hwan Oh and Dong Nyung Lee

Department of Metallurgical Engineering and Center for Advanced Materials Research,
Seoul National University, Seoul 151-742, Korea

Pohang P.O. Box 36, 1, Koedong-dong, Pohang-shi, 790-785, Korea

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Introduction

The proper designing and manufacture of tube sheets in heat exchangers and shadow masks in color picture tubes require the characterization of the deformation behavior of perforated sheets containing a large number of holes. The analysis of the stress and strain in the perforated material is based on treating the perforated material as an equivalent solid material [1–3]. Many attempts have been made to characterize deformation behavior of perforated sheets [3–13]. The apparent elastic constants of the perforated sheets were calculated [3,5]. The yield criteria of perforated sheets with a uniform triangular pattern of round holes were proposed [7,9,14]. When the perforated sheet is formed into an appropriate shape, difficulties arise in control of variables for the optimum fabrication because of many holes [10]. The limit or failure strains in sheet metal forming are best represented by a forming limit curve (FLC) which indicates the onset of necking over all possible combinations of strains in the plane [15,16]. A number of theoretical treatments have been developed to predict the FLC of continuum materials. However, there are few reports on the experimental and theoretical studies on FLC for a perforated sheet [17]. The purpose of this paper is to propose a model which can explain the FLC of a perforated sheet for dot-type shadow mask.

Calculation of FLC of a Perforated Sheet

Fig. 1 shows the perforated sheet with a large number of closely spaced circular holes arranged in an equilateral triangular pattern. When the perforated sheet is loaded, yielding begins in the ligament or the zone between the nearest neighboring holes and the strain in the ligament is larger than any other part [5,7–9]. The onset of necking is generally accepted as a criterion of failure [14,15], and the FLC of the perforated sheet can be obtained from the onset of necking of the ligament over all possible combinations of apparent strains in the plane of the sheet. The plastic strain increments of base metal in the ligament depend on the apparent plastic strain increments. Let two normal strain increments in the s and n directions and one shear strain increment of base metal in the ligament in Fig. 1 be de, ds, and dm and the apparent strain increments be de*, ds*, and dm*. The strains can be obtained by the finite element analysis using updated Lagrangian formulation with von-Mises yield function as follows: Fig. 2 shows the mesh used in the elasto-plastic finite element analysis for stretching of a perforated sheet with uniform holes. The boundary conditions for stretching in the s and n
Figure 1. Hexagonal array of holes in the perforated sheet.

Figure 2. Initial finite element mesh for 3-dimensional calculation of deformation behaviour of the perforated sheet, where the ligament strains have been obtained from the hatched element.

directions are given in Table 1. \( U_s \) and \( U_n \) in Table 1 denote the prescribed displacements in the \( s \) and \( n \) directions, respectively. The apparent strains, \( \varepsilon_s^* \) and \( \varepsilon_n^* \), are calculated using the following equations.

\[
\varepsilon_s^* = \ln\left(\frac{s_c}{s_{co}}\right)
\]

\[
\varepsilon_n^* = \ln\left(\frac{n_d}{n_{do}}\right)
\]

where \( n_{do} \) and \( n_d \) are the lengths of OA before and after deformation, whereas \( s_{co} \) and \( s_c \) are the lengths of OC before and after deformation.

The strains in the ligament, \( \varepsilon_s \) and \( \varepsilon_n \), can be obtained by averaging the strains in the hatched elements in Fig. 2. Fig. 3 shows calculated \( \varepsilon_s \) and \( \varepsilon_n \) as functions of \( \varepsilon_s^* \) and \( \varepsilon_n^* \). The strains in the ligament may be approximated by linear functions of the apparent strains as follows:

\[
d\varepsilon_s = k_{sd}d\varepsilon_s^* + k_{nd}d\varepsilon_n^*
\]

\[
d\varepsilon_n = k_{md}d\varepsilon_s^* + k_{nd}d\varepsilon_n^*
\]

### TABLE 1
Boundary Conditions of 3D FEM

<table>
<thead>
<tr>
<th>stretching direction</th>
<th>plane OO'A'A</th>
<th>plane AA'B'B</th>
<th>plane BB'C'C</th>
<th>plane CC'O'O</th>
</tr>
</thead>
<tbody>
<tr>
<td>s-axis</td>
<td>( U_s = 0 )</td>
<td>( U_n = 0 )</td>
<td>( U_s = \text{specified} )</td>
<td>( U_n = 0 )</td>
</tr>
<tr>
<td>n-axis</td>
<td>( U_s = 0 )</td>
<td>( U_n = \text{specified} )</td>
<td>( U_s = 0 )</td>
<td>( U_n = 0 )</td>
</tr>
<tr>
<td>s and n axes</td>
<td>( U_s = 0 )</td>
<td>( U_n = \text{specified} )</td>
<td>( U_s = \text{specified} )</td>
<td>( U_n = 0 )</td>
</tr>
<tr>
<td>z-axis</td>
<td></td>
<td></td>
<td>( U_n/U_s = 1/2, 1, 2 )</td>
<td>Traction free (( T_z = 0 ))</td>
</tr>
</tbody>
</table>
For biaxial stretching, $\epsilon_s$, $\epsilon_n$ and $\epsilon_m$ can be expressed in terms of $\epsilon_x$, $\epsilon_y$ and $\theta_{jr}$ as follows:

$$d\epsilon_s, d\epsilon_n = A d\epsilon_x, d\epsilon_y$$

where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The effective plastic strain increment of each ligament can be calculated using equations (4) and (5).

It is proposed that necking of the ligament of a perforated sheet occurs when the effective plastic strain of the ligament is equal to a critical strain, $\epsilon_c$. There are the vertical ligament ($\theta_{jr} = 90^\circ$) and the diagonal ligament ($\theta_{jr} = \pm 30^\circ$) in the perforated sheet shown in Fig. 1. For biaxial stretching, the forming limit may be defined by an apparent strain state so that the larger one of the effective strains in the two ligaments at $\theta_{jr} = 90^\circ$ and $\theta_{jr} = \pm 30^\circ$ can reach $\epsilon_c$. Thus, FLC of the perforated sheet consists of forming limits obtained for the various biaxiality ratios. The $\epsilon_c$ may be defined as the effective plastic strain of ligament at the maximum load of a uniaxial tension in the $x$ direction of the perforated sheet.

The effective strain of ligament under unaxial tension is calculated by equations (4) and (5). Let $x'$ and $y'$ denote the axes in the uniaxial tension and width directions, respectively. The apparent strain increments, $d\epsilon_x$, $d\epsilon_y$, and $d\epsilon_{xy}$, can be obtained using equation (3), where $d\epsilon_x$, $d\epsilon_y$ and $d\epsilon_{xy}$ are replaced by $d\epsilon_x'$, $d\epsilon_y'$ and $d\epsilon_{xy}'$. The shear strain in the center of the ligament, $\epsilon_{xy}$ and $\epsilon_{yx}$, can be neglected in the in-plane stretching. For von Mises material, the effective plastic strain increment in the center of the ligament, $d\epsilon_p$, can be obtained from equations (2) and (3) as follows:

$$d\epsilon_p = d\epsilon_{xy}'$$

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$$d\epsilon_p = d\epsilon_{xy}'$$
\[
d \varepsilon_{il} = \frac{2}{\sqrt{3}} \left[ (k_{ss}^2 + k_{sn} k_{ns} + k_{nn}^2) \varepsilon_s^2 + (2k_{sn} k_{ss} + k_{ss} k_{nn} + k_{nn} k_{sn} + 2k_{st} k_{nt}) \right] d \varepsilon_s^2 + \left( k_{sm} + k_{sn} k_{nm} + k_{nm}^2 \right) d \varepsilon_s^2 = \frac{2}{3} \left( k_{sm} + k_{sn} k_{nm} + k_{nm}^2 \right) d \varepsilon_s^2 + \frac{2}{3} \left( k_{sm} + k_{sn} k_{nm} + k_{nm}^2 \right) d \varepsilon_s^2 \right]
\]

(4)

For biaxial stretching, \( \varepsilon_s^*, \varepsilon_n^* \) and \( \varepsilon_m^* \) can be expressed in terms of \( \varepsilon_s, \varepsilon_n, \varepsilon_m \) and \( \psi \) as follows:

\[
\begin{bmatrix}
\varepsilon_s^* \\
\varepsilon_n^* \\
\varepsilon_m^*
\end{bmatrix} = A \begin{bmatrix}
\varepsilon_s \\
\varepsilon_x \\
\varepsilon_y
\end{bmatrix} A^T \begin{bmatrix}
\cos \psi & 0 & -\sin \psi \\
0 & 1 & 0 \\
-\sin \psi & 0 & \cos \psi
\end{bmatrix}
\]

(5)

where \( \psi \) is the angle between \( x \) and \( s \) axes of the ligament in Fig. 1. The effective plastic strain increment of each ligament can be calculated using equations (4) and (5).

It is proposed that necking of the ligament of a perforated sheet occurs when the effective plastic strain of the ligament is equal to a critical strain, \( \bar{\varepsilon}_c \). There are the vertical ligament \( (\psi = 90^\circ) \) and the diagonal ligament \( (\psi = \pm 30^\circ) \) in the perforated sheet shown in Fig. 1. For biaxial stretching, the forming limit may be defined by an apparent strain state so that the larger one of the effective strains in the two ligaments at \( \psi = 90^\circ \) and \( \psi = \pm 30^\circ \) can reach \( \bar{\varepsilon}_c \). Thus, FLC of the perforated sheet consists of forming limits obtained for the various biaxiality ratios. The \( \bar{\varepsilon}_c \) may be defined as the effective plastic strain of ligament at the maximum load of a uniaxial tension in the \( x \) direction of the perforated sheet.

The effective strain of ligament under uniaxial tension is calculated by equations (4) and (5). Let \( x' \) and \( y' \) denote the axes in the uniaxial tension and width directions, respectively. The apparent strain increments, \( d\varepsilon_{s}', d\varepsilon_{n}', d\varepsilon_{m}' \), can be obtained using equation (3), where \( d\varepsilon_{s}', d\varepsilon_{n}', d\varepsilon_{m}' \) are replaced by \( d\varepsilon_{x}', d\varepsilon_{y}' \) and \( d\varepsilon_{xy}' \) and \( \psi \) is the angle between \( x' \) axis and \( x \) axis of the ligament. Substitution of \( d\varepsilon_{s}', d\varepsilon_{n}', d\varepsilon_{m}' \) into equation (4) gives the effective strain increment of each ligament. A critical effective plastic strain of ligament, \( \bar{\varepsilon}_c \), is one which gives rise to the maximum load.

**Experimental Methods**

A commercially available perforated sheet for dot-type shadow mask was used as the perforated sheet for this study. Fig. 5 shows the perforated sheet for dot-type shadow mask and the cross section of a hole whose size varies along the thickness direction. The average diameter of hole was 0.214mm. The previous work [18] showed that the perforated sheet for shadow mask behaved as if it were the sheet having homogeneous holes.

![Figure 5. Scanning electron micrograph of the perforated sheet for dot-type shadow mask and the geometry of hole.](image-url)
whose diameter was calculated by averaging hole sizes along thickness. The base metal of the sheet was Fe-36Ni Invar, whose Young's modulus, the yield stress, the plastic modulus and Poisson’s ratio are 151GPa, 232MPa, 1280MPa and 0.280 [12], respectively. Uniaxial tensile tests of the perforated sheet were performed. Tensile specimens with a gauge length of 25mm and a width of 6mm were cut from the perforated sheet at 0°, 22.5°, 45°, 67.5° and 90° to x axis. The specimens were tested at a constant cross head speed of 3.4 mm min⁻¹. In the uniaxial tension of perforated sheet, the apparent limit strain was obtained by measuring the apparent strain at the maximum load.

The FLC was obtained from in-plane punch stretching experiments in Fig. 6. The punch stretching experiment consists of clamping sheets over a lockbead and stretching them to failure using a 50 mm diameter cylindrical punch. The perforated sheet was clamped between two 0.8mm thick brass sheets containing a 30mm diameter hole to prevent from failure near bead. The perforated sheet specimens were 50mm in length, whose axial direction is parallel or vertical to x axis, and had different widths from 5 to 50mm. The brass sheets were 100mm in length and had different widths from 60 to 100mm. After stretching, the failure strains in the x and y directions (apparent) were obtained by measuring the distance between two equivalent points containing one hole near the cracked area.
Figure 9. Optical micrographs showing necking of (a) the vertical and (b) diagonal ligaments.

Results and Discussion

The best fitting of the calculated results in Fig. 3 by equation (2) yields $k_{x}$, $k_{y}$, $k_{m}$ and $k_{n}$ being $-0.209$, $0.428$, $-0.877$ and $1.677$, respectively.

Under uniaxial tension, the specimen failed immediately after the maximum load, that is, the apparent uniform elongation was almost the same as the apparent failure strain. Since the perforated sheet has 12 symmetry lines, the apparent deformation is almost isotropic and the apparent shear strain can be neglected in the uniaxial tension. Fig. 7 shows the $e_{y}' - e_{y}'$ relations under uniaxial tension in various directions, which can be best fitted to $e_{y}' = -0.5 e_{x}'$. Under uniaxial tension, the apparent limit strain in the tension direction, $e_{m}'$, can be calculated using equations (4), (5) and the relation of $e_{y}' = -0.5 e_{x}'$. Fig. 8 represents the calculated and measured $e_{m}'$ as a function of angle between $x'$ axis and $x$ axes. This figure shows that the calculated results are in good agreement with the experimental data. For arbitrary stretching of the perforated sheet, the strain of one ligament (for example, the vertical ligament) can be higher than that of the other ligament (the diagonal ligament), and the ligament of higher strain will fail.

Figure 10. FLC of the perforated sheet for dot-type shadow mask.