Analysis of coefficient of friction in compression of porous metal rings

H. N. Han, H. S. Kim, K. H. Oh, and D. N. Lee

The elastoplastic finite element method using a yield criterion advanced by Lee and Kim was employed to analyse the compression of sintered porous metal rings. Densities of the porous rings and compression loads have been calculated as a function of height reduction. The changes in geometry of porous metal rings with initial relative density were also calculated for various coefficients of friction between the metal rings and compression plates. The coefficient of friction could be determined from the relationship between the change in the inner diameter and height reduction of porous metal rings with various initial relative densities. From the changes in the average relative densities of the porous metal rings, evaluation of lubricants could be also conducted.

An understanding of the deformation behaviour and the density distribution of porous metals during forming is very important in the production of good quality powder metallurgical parts. In the forming of sintered porous metals, the elastic-plastic deformation behaviour is influenced by the internal pores, and the friction conditions will be different from those in the conventional forming of non-porous solid metals. Therefore, in order to find a proper lubricant for forming sintered porous metals, a method for the evaluation of lubricants must be established.

The ring compression test is widely applicable to evaluate lubricants, because this method only requires the measurement of the change in geometry of a ring. Many experimental and theoretical studies on the ring compression testing of conventional non-porous solid metals have been undertaken by various investigators. Tabata and Masaki have carried out and analysed the compression of sintered porous copper rings for determining the friction coefficient between the workpieces and compression plates using the stress analysis method and Oyane's yield criterion modified by Tabata et al. Oh and Mun have carried out and analysed the ring compression of sintered porous iron rings using the upper bound approach for determining the friction factor. Oh and Mun have used another yield criterion due to Oyane. The above two studies are based on the following assumptions:

(i) bulging does not take place at the inner and outer surfaces of the porous metal rings
(ii) the material is rigid, perfectly plastic, and does not work harden.

Lee and Kim have reviewed yield criteria for porous metals and found that most of them unreasonably suggest zero yield stress only at zero relative density.

Lee and Kim modified a yield equation suggested by Doraiswamy et al., so that it could incorporate one parameter which could be estimated from the yield stress versus initial relative density data. The modified yield condition for porous metals may be expressed as

\[
(2 + R^2)J_2 + (1 - R^2)J_1^2/3 = \eta Y_0
\]

where

\[
\eta = Y_0/\gamma_0 = [(R - R_c)(1 - R_c)]^2
\]

\[
J_2 = \frac{1}{2}(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2
\]

\[
J_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}
\]

In the above equations, \(J_2\) and \(J_1\) are the quadratic stress and the linear stress invariant, respectively, \(Y_0\) is the yield stress of porous metal with relative density \(R\), and \(Y_0\) is the yield stress at \(R = 1\). For work hardening metals, \(Y_0\) and \(Y_0\) are the flow stresses of the porous and non-porous metals. The parameter \(R_c\) is an experimental parameter and may be interpreted as a critical relative density at which the yield stress of porous metal becomes zero: that is, \(Y_0 = 0\) at \(R = R_c\). For non-porous metals, \(R = 1\) and \(Y_0 = Y_0\) and equation (1) becomes the Von Mises yield criterion, \(3J_2 = Y_0^2\). Equation (1) could successfully simulate the densification process of porous iron specimens with various initial porosities under hydrostatic pressure. Han et al. formulated the elastoplastic finite element analysis of deformation of porous metals using equation (1), and were able to satisfactorily predict the deformation behaviours in simple upsetting and indenting of sintered porous metals.

In the present work, the analysis of the coefficient of friction in the compression of sintered porous metal rings has been made based on the elastoplastic finite element method using equation (1). The changes in geometries of porous metal rings with various initial relative densities have been calculated for various coefficients of friction. The distribution of relative density and the change of average relative density of porous metal ring were also calculated for various coefficients of friction.

ELASTOPLASTIC FINITE ELEMENT FORMULATION

For the yield condition of equation (1), under the assumptions of isothermal conditions and isotropic hardening, the yield function \(F\) at time \(t\) can be expressed as

\[
F = (2 + R^2)J_2^2/3 + (1 - R^2)J_1^2/3 + \eta Y_0^2
\]

where \(Y_0\), \(R\) and \(\eta\) are state variables dependent on the plastic strains \(\varepsilon_p\). The yield function \(F\) is used to calculate the plastic strain increments as follows:

\[
d\varepsilon_p = \frac{1}{2} \frac{\partial F}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \varepsilon_p} = \frac{1}{2} q_0 C E d\varepsilon_p + \frac{1}{2} q_0 C E d\varepsilon_p
\]

\[
d\varepsilon = C_{\varepsilon} (d\varepsilon_p - d\varepsilon) = C_{\varepsilon} (d\varepsilon_p - d\varepsilon) = \left[ C_{\varepsilon} - \frac{1}{2} q_0 C E \left(q_0 C E\right)\right] d\varepsilon_p
\]
where \( i \) is a positive scalar, \( C^E \) is an elastic stress-strain matrix, \( C^{EP} \) is the relative density dependent elastoplastic stress-strain matrix, and \( 'p^T \) and \( 'q^T \) are transposes of \( p^T \) and \( q^T \), which are defined by the following equations

\[
't_E^T = \begin{bmatrix} 'p_{11}'p_{22}'p_{33}'p_{12}'p_{13}'p_{23} \end{bmatrix} \quad \ldots \quad (5)
\]

\[
'y^T = \begin{bmatrix} 'q_{11}'q_{22}'q_{33}'q_{12}'q_{13}'q_{23} \end{bmatrix} \quad \ldots \quad (6)
\]

\[
't_{ij} = -\delta F/\delta e_{ij} \quad \text{and} \quad 'q_{ij} = \delta F/\delta \sigma_{ij} \quad \ldots \quad (7)
\]

The elastic stress-strain matrix \( C^E \) is given by

\[
C^E = \frac{E(1-v)}{(1+v)(1-2v)} \begin{bmatrix}
1 & v & v & 0 & 0 & 0 \\
v & 1 & v & 0 & 0 & 0 \\
v & v & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)}
\end{bmatrix} \quad \ldots \quad (8)
\]

Equations (3–8) are also valid in the case of non-porous materials.\(^{18}\) The relationship between the apparent plastic deformation energy increment per unit volume of porous metal \( d\mathcal{W} \) and that of the non-porous base metal \( d\mathcal{W}_0 \) is given by

\[
d\mathcal{W} = R' d\mathcal{W}_0 \quad \text{or} \quad '\sigma_{ij} d\varepsilon_{ij} = R' \sigma_{ij} d\varepsilon_{ij} \quad \ldots \quad (9)
\]

where \( d\varepsilon_{ij} \) is the effective plastic strain increment of the non-porous base metal. Since the yield stress is a function of the plastic work per unit volume, \( 'p_{ij} \) and \( 'q_{ij} \) may be evaluated using the following equations

\[
'p_{ij} = \frac{2}{3} R' \sigma_{ij} \quad \ldots \quad (10)
\]

\[
'q_{ij} = \frac{2+R'2}{3} \sigma_{ij} + \frac{2}{9} (1-R'2)J_1 \delta_{ij} \quad \ldots \quad (11)
\]

where \( E', E_T, '\sigma_{ij} \), and \( \delta_{ij} \) are the elastic modulus and tangential modulus of the non-porous base metal, deviatoric stresses, and the Kronecker delta respectively. For non-porous metals, \( R' = 1 \) and \( 'q = 1 \), and equations (10) and (11) become those of the Von Mises yield condition. The elastoplastic stress-strain matrix \( C^{EP} \) and the positive scalar \( \mu \) can be formulated using equations (3), (4), (8), (10), and (11).

### UPDATING RELATIVE DENSITY DURING DEFORMATION

The constancy of mass during deformation of porous metals gives the following relationship

\[
R = R_0 V_0 / V \quad \ldots \quad (12)
\]

where \( R_0 \) and \( R \) are the relative density before and after deformation, and \( V_0 \) and \( V \) are the volume before and after deformation. For an axisymmetric body, the volume of material associated with an element of area \( A \) that is that of a body of revolution of the element and the volume is given by

\[
V = \int_A 2\pi x \, dA \quad \ldots \quad (13)
\]

where \( x \) is the distance between the centroid of the element and the axisymmetric line. The first step to calculate volume \( V \) is to relate the actual global coordinates \( x \) and \( y \) to a natural coordinate system \((-1 \leq \xi \leq 1, -1 \leq \eta \leq 1)\).

The integration of a scalar function \( f(\xi, \eta) \) in the natural coordinate can be obtained by applying the integration formula successively, namely

\[
\int_{-1}^{1} \int_{-1}^{1} f(\xi, \eta) \, d\xi \, d\eta = \sum_{m=1}^{m} \sum_{n=1}^{n} w_i w_j f(\xi_i, \eta_j) \quad \ldots \quad (14)
\]

where \( w_i \) and \( w_j \) are weight factors, and \( m \) and \( n \) are the order of integration. For a function \( f(x, y) = 2\pi x \) defined over an isoparametric element, integration can be evaluated as

\[
V = \int_{-1}^{1} \int_{-1}^{1} f(\xi, \eta) |J| \, d\xi \, d\eta
\]

where \( |J(\xi_i, \eta_j)| \) is the determinant of a Jacobian matrix and is given by

\[
|J(\xi_i, \eta_j)| = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \xi} \quad \ldots \quad (15)
\]

The updated relative density \( R \) can be obtained using equations (12), (15), and (16).
ANALYSIS OF RING COMPRESSION OF POROUS METALS

Assuming homogeneous deformation in compression of porous metal rings, the change in the relative density can be expressed as a change in volume

\[
\frac{dR}{R} = \frac{-dV}{V} = -\left[2\left(r_o dr_o - r_i dr_i\right) + \frac{dh}{h}\right]
\]

where \(r_o, r_i\), and \(h\) are outer and inner radii and height of porous metal rings during compressing. Poisson’s ratio in ring compression can be given as

\[
v = -\left(\frac{dr_o - dr_i}{r_o - r_i}\right)\left(\frac{dh}{h}\right)
\]

The relationship between \(v\) and relative density \(R\), suggested by Kuhn, is given by

\[
v = 0.5R^2
\]

In fact, this relationship was used to derive the yield condition of equation (1). From the assumption of homogeneous deformation, the radial strain increment \(d\varepsilon_{rr}\) of the porous metal ring in outer and inner region is given by

\[
d\varepsilon_{rr} = \frac{dr_o}{r_o} = \frac{dr_i}{r_i}
\]

It follows from equations (17) to (20) that

\[
\frac{dR}{(R^2 - 1)} = \frac{dh}{h}
\]

The relative density varies from \(R_0\) to \(R\), as the height of the metal ring varies from \(h_0\) to \(h\). Integration of equation (21) gives

\[
R = \left[\left(\frac{h_0}{h}\right)^2 + 1 - \frac{R_0^2}{R_0^2}\right]^{1/2}
\]

From the constancy of mass during deformation of porous metals, this relationship may be rewritten as

\[
(r_o^2 - r_i^2)h_R = (r_o^2 - r_i^2)h_0R_0
\]

where \(r_0\) and \(r_i\) are the initial outer and inner radii, respectively. The inner radius of a porous metal ring during compression can be evaluated using equations (20) and (23) as follows

\[
r_i = (r_o^2h_0R_0/h_R)^{1/2}
\]

The above equation implies that the volume of the central hole through the ring is a function of the density of the metal. In other words, for homogeneous deformation, metals in the form of porous rings behave as if they are central holeless porous metals.

RESULTS AND DISCUSSION

Oh and Mun measured the average relative density and change in the inner diameter of sintered porous iron rings which were subjected to compression under various friction conditions. They used specimens of two different sizes with dimensions of 28 mm in outer diameter (2\(r_o\)), 14 mm in inner diameter (2\(r_i\)), and 6 mm in height, namely, 2\(r_o\)/2\(r_i\) = 6:3:1; and 24 mm in outer diameter, 12.5 mm in inner diameter and 6 mm in height, namely, 2\(r_o\)/2\(r_i\) = 6:3:1:5. Their experimental results are shown in Fig. 1, along with calculated results which will be explained below.

The mechanical properties (\(\sigma_{eq}\) in MN m\(^{-2}\)) of non-porous iron (\(\nu = 0.3, E = 209 G\text{N} m^{-2}\)) were taken to be \(\sigma_{eq} = 6600\sigma_{eq}^0\) and \(E = 209 G\text{N} m^{-2}\) for the finite element calculation of grid distortions, density distributions, and changes in geometry of porous rings. The initial grid for ring compression is shown in Fig. 2. The finite element method calculation results are shown in Fig. 1 along with the measured data. The friction coefficients \(\mu\) in the calculation were set to best fit the measured data. Figure 1 demonstrates that the elastoplastic finite element method using the yield condition of equation (1) simulates the deformation of the porous metal rings quite closely. For reference, grid distortion, relative density, compression load, and average density during deformation of the porous iron specimens have been calculated.

![Graph showing calculated grid distortions in porous iron rings](image)

3 Calculated grid distortions in porous iron rings (2\(r_o\)/2\(r_i\)/\(h_0\) = 6:3:1) with different relative densities 0.75, 0.85, and 0.95, reduced by 50% under various friction conditions.
Figure 3 shows the grid distortions of porous iron rings \((2r_o/2r_i/h_o = 6:3:1)\) with various initial relative densities reduced by 50\% under various friction conditions. At lower levels of friction, the inner and outer surfaces flow outward, resulting in outward barrelling. As friction increases, a neutral flow point, where the radial velocity of the deforming material becomes zero, appears in the specimen, resulting in the inward barrelling of the inner surface and the outward barrelling of the outer surface. At lower levels of friction, \(\mu = 0.02\) and 0.05, no folding occurs, whereas at a higher level of friction, \(\mu = 0.2\), folding occurs at the inner edge as well as the outer edge.

Figure 4 shows the relative density distributions in porous iron rings \((2r_o/2r_i/h_o = 6:3:1)\) with an initial relative density of 0.85 reduced by 20, 30 and 50\% under various friction conditions. At the low coefficient of friction of 0.02, the deformation is relatively homogeneous, and hence the difference between maximum and minimum relative densities observed at regions A and B, respectively is small (<0.02). At the higher coefficient of friction of 0.2, the least densification is observed near the region B due to a large outward barrelling. The highest densification occurs near the inner edge of the die–specimen contact area for a height reduction of 20\%, but the highest densification occurs in the middle region of the porous metal ring at higher reductions. Figure 5 shows the changes of relative densities at the equatorial inner and outer elements (regions A and B in Fig. 2) for a porous iron ring \((2r_o/2r_i/h_o = 6:3:1)\) with height reduction. For a low coefficient of friction of 0.02, relative densities in the regions A and B increase as the height reduction increases. But, for a high coefficient of friction of 0.2, relative densities in the regions A and B increase to maxima and decrease due to the large inward and outward barrellings. The relative density in region A is always higher than that in region B regardless of the coefficient of friction, and the difference between the relative densities in regions A and B decreases owing to the increasing homogeneity of deformation with decreasing coefficient of friction.

Figures 6 and 7 show the load–height reduction curves and the average relative density–height reduction curves respectively, of porous iron rings \((2r_o/2r_i/h_o = 6:3:1)\) of an initial relative density of 0.85 with \(\mu = 0.0, 0.02, 0.05, 0.1, \) and 0.2. In the upsetting of cylindrical porous metals, the forming load and average relative density were not very sensitive to friction conditions. But in ring compression, the forming load and the average relative density show a notable increase with increasing coefficient of friction. An increase in the coefficient of friction makes outward sliding of the ring more difficult (Fig. 4), resulting in greater densification, which in turn requires a greater load for a given reduction. It is noted that the finite element solutions for frictionless compression and equation (22)
6 Calculated compression load as function of height reduction of porous iron ring \((2r_{o0}/2r_{o0})/h_0 = 6:3:1\) with initial relative density of 0.85 under various friction conditions

\[\sigma_{eq} = 946(e_{eq} + 0.003)^{0.5833} \quad \text{for} \quad 0 < e_{eq} < 0.034 \]

\[\sigma_{eq} = 482(e_{eq} - 0.0105)^{0.333} \quad \text{for} \quad e_{eq} > 0.034 \]

\[R_c = 0.442\]

7 Calculated average relative density of porous iron ring \((2r_{o0}/2r_{o0})/h_0 = 6:3:1\) with initial relative density of 0.85 as function of height reduction under various friction conditions; symbols and solid curve are values calculated by finite element method and equation (22) respectively.
The finite element solution and the analytical solution (equation (24)) under frictionless compression are also seen to give identical results. These figures indicate that the evaluation of lubricants which are frequently used in the forming of porous metals can be carried out.

Figure 9a–c shows the relationship between the change in inner diameter and the height reduction for porous ring specimens with various initial relative densities and geometries compressed under conditions of a given coefficient of friction. From these figures, the coefficient of friction can be obtained from measurement of the deformed ring geometry. For a large coefficient of friction, the rate of decrease in the inner diameter of the porous metal rings increases with increasing initial relative density. For a small coefficient of friction, the rate of increase in the inner diameter increases with increasing initial relative density. This means that the overall change in the ring geometry decreases and the densification of the ring increases with decreasing initial relative density.

CONCLUSIONS
An analysis of ring compression of sintered porous metals was made on the basis of the elastoplastic finite element method using a yield criterion advanced by Lee and Kim.\(^{10}\) This has enabled ring compression testing to be used as a method of determining the coefficient of friction in porous metal forming processes. The evaluation of lubricants for forming porous metals was conducted by comparing the experimental data with finite element method solutions. The average relative density during ring compression increases with increasing friction, and lubricants can be evaluated by measurement of the change of the average relative density and the change of inner diameter. The rate of decrease in the inner diameter for a large coefficient of friction and the rate of increase for a small coefficient of friction increase with increasing initial relative density.

ACKNOWLEDGEMENT
This work has been supported by the Korea Science and Engineering Foundation through the Research Center for Thin Film Fabrication and Crystal Growing of Advanced Materials, Seoul National University, Seoul, Korea.

REFERENCES

Changes calculated by finite element method in geometry of porous metal rings of given initial geometry.