Elastoplastic finite element analysis for porous metals

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The elastoplastic finite element method for the deformation of porous metals has been newly formulated using the yield condition advanced by Lee and Kim. Changes in geometries and densities of porous metals, and upsetting loads with upsetting strain have been calculated. The Brinell hardnesses of porous metals with various densities dependent on indenting geometries have been measured. The calculated results were in very good agreement with the measured data.

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Sintered products are often subjected to sizing or forged to final products. Therefore, understanding the deformation behaviour of porous metals during forming is very important in achieving good quality powder metallurgical parts. Deformation behaviour of porous metals has been analysed by many investigators using slip line field theory, limit analysis, and rigid-plastic finite element analysis. The elastoplastic deformation behaviour of porous metals is influenced by the internal pores. Analysis of deformation behaviour of porous metals requires an appropriate yield criterion which should take the pore effect into account. Lee and Kim have reviewed yield criteria for porous metals and found that most of them unreasonably suggest zero yield stress only at zero relative density. Lee and Kim modified a yield equation suggested by Doraivelu et al. so that it could incorporate one parameter which could be estimated from the yield stress versus initial density data. The modified yield condition for porous metals may be expressed as

\[(2 + R^2)J_2 + (1 - R^2)J_1/3 = \eta Y_0^2\]  

where

\[\eta = Y_0^2 / Y_0^3 = [(R - R_c)(1 - R_c)]^{1/3}\]

\[J_2 = \frac{1}{8}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + \sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2\]

\[J_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}\]

In the above equations, \(J_2\) and \(J_1\) are the quadratic stress deviator invariant and the linear stress invariant, respectively. \(Y_0\) is the yield stress of porous metal with relative density \(R\), and \(Y_0\) is the yield stress at \(R = 1\). For work hardening metals, \(Y_0\) and \(Y_0\) are the flow stresses of the porous and non-porous metals. The parameter \(R_c\) is an experimental parameter and may be interpreted as a critical relative density in which the yield stress of porous metal becomes zero; that is, \(Y_0 = 0\) at \(R = R_c\). For non-porous metals, \(R = 1\) and \(Y_0 = Y_0\). Equation (1) becomes the Von Mises yield criterion, \(3J_2 - Y_0^2\). Equation (1) could successfully simulate the densification process of porous iron specimens with various initial porosities under hydrostatic pressure. The purpose of the present work is to examine how the yield equation, equation (1), predicts various plastic deformations of porous metals and to formulate the elastoplastic finite element analysis of the deformation of porous metals using this equation.

ELASTOPLASTIC FINITE ELEMENT FORMULATION

For the yield condition of equation (1), under the assumptions of isothermal conditions and isotropic hardening, the yield function \(F\) at time \(t\) can be expressed

\[F = (2 + R^2)J_2/3 + (1 - R^2)J_1/3 - \eta Y_0^2\]  

where \(Y_0\) and \(\eta\) are state variables dependent on the plastic strains \(\varepsilon^p\). The yield function \(F\) is used to calculate the plastic strain increments as follows

\[d\varepsilon_p = \frac{1}{2} \frac{\partial F}{\partial \varepsilon_p} \]  

\[d\varepsilon_p = C^p (d\varepsilon_p - d\varepsilon_0) = C^p d\varepsilon_0 = \left[ C^p - \frac{1}{2} \frac{\partial C^p}{\partial \varepsilon_p} \right] d\varepsilon_0 \]

where \(\frac{1}{2}\) is a positive scalar, \(C^p\) is an elastic stress-strain matrix, \(C^p\) is the relative density dependent elastoplastic stress-strain matrix, and \(\varepsilon^p\) and \(\varepsilon_0\) are transposes of \(\varepsilon^p\) and \(\varepsilon_0\), which are defined by equations (5-7).

The elastic stress-strain matrix \(C^p\) is given by

\[C^p = \frac{E(1 - v)}{(1 + v)(1 - 2v)} \left[ \begin{array}{ccc} \frac{v}{1 - v} & \frac{v}{1 - v} & 0 \\ \frac{v}{1 - v} & \frac{v}{1 - v} & 0 \\ 1 & 1 & 0 \end{array} \right] \times \left[ \begin{array}{ccc} \frac{1 - 2v}{2(1 - v)} & 0 & 0 \\ 0 & \frac{1 - 2v}{2(1 - v)} & 0 \\ \frac{1 - 2v}{2(1 - v)} & 0 & \frac{1 - 2v}{2(1 - v)} \end{array} \right] \]

Equations (3-8) are also valid in the case of non-porous materials. The relationship between the apparent plastic deformation energy increment per unit volume of porous metal \(d\tilde{W}\) and that of the non-porous base metal \(dW_0\) is
given by

\[ dW = R \, dW_0 \quad \text{or} \quad \delta_{ij} \, d\varepsilon_{ij} = R \, \delta_{ij} \, d\varepsilon_{ij} \]  

where \( d\varepsilon_{ij} \) is the effective plastic strain increment of nonporous metal. Since \( d\varepsilon_{ij} \) is the strain increment of the plastic work per unit volume, we can evaluate \( \delta_{ij} \) and \( \delta_{ij} \) using the following equations

\[ \delta_{ij} = \frac{2 + R^2}{3} \delta_{ij} + \frac{2}{9} (1 - R^2) J \delta_{ij} \]  

(10)

where \( E, \sigma_{ij}, \) and \( \delta_{ij} \) are the elastic modulus, tangential modulus of non-porous base metal, deviatoric stress, and the Kronecker delta respectively. For nonporous metals, \( R = 1 \) and \( \epsilon = 1 \), and equations (10) and (11) become those of the von Mises yield condition. The elastoplastic stress–strain matrix \( C_{EP} \) and the positive scalar \( \psi \) can be formulated using the equations (3), (4), (8), (10), and (11).

**UPDATING RELATIVE DENSITY**

The constancy of mass during deformation of porous metals is given the following relationship

\[ R = R_0 \frac{V}{V_0} \quad \text{or} \quad \rho \, V = \frac{\rho_0}{\rho_0} \frac{V}{V_0} \]  

(12)

where \( R_0 \) and \( R \) are the relative density before and after deformation, and \( V_0 \) and \( V \) are the volume before and after deformation. The volume of an axisymmetric element is given by

\[ V = \int_A \frac{2\pi x}{3} \, dA \]  

(13)

where \( x \) is the distance between the centroid of the element and the axisymmetric line, and \( A \) is the area of the element. The first step to calculate volume \( V \) is to relate the actual global coordinates \( x \) and \( y \) to a natural coordinate system \((-1 \leq \xi \leq 1, -1 \leq \eta \leq 1)\).

\[ x = \xi, \quad y = \eta \]  

(14)

\[ \frac{dx}{d\xi} \, \frac{dy}{d\eta} \]  

\[ \frac{dx}{d\xi} = \frac{\partial x}{\partial \xi} \quad \frac{dy}{d\eta} = \frac{\partial y}{\partial \eta} \]  

(15)

The integration of a scalar function \( f(\xi, \eta) \) in the natural coordinate can be obtained by applying the integration formula successively, namely

\[ \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} f(\xi, \eta) \, d\xi \, d\eta \]  

(16)

where \( w_i \) and \( w_j \) are the weight factors, and \( n \) and \( m \) are the order of integration. For a function \( f(x, y) = 2\pi x \) defined over an isoparametric element, integration can be evaluated as

\[ V = \sum_{i=1}^{n} \sum_{j=1}^{m} w_i \, w_j \, f(\xi_i, \eta_j) \, J(\xi_i, \eta_j) \]  

(17)

where \( J(\xi_i, \eta_j) \) is the determinant of a Jacobian matrix and is given by

\[ J(\xi_i, \eta_j) = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \xi} \]  

(18)

The updated relative density \( R \) can be obtained using equations (12), (17), and (18).

**APPLICATIONS**

**Simple upsetting**

Shima and Oyane performed frictionless upsetting of 20 mm diameter, 25 mm high sintered cylindrical copper specimens with various relative densities. Fig. 1 shows the variation of \( Y_B \) with \( R \) for the specimens. The mechanical properties of the non-porous copper specimen \((\nu = 0.3, \, E = 117 \, \text{GN m}^{-2}, \, \sigma_0 = 54 \, \text{MN m}^{-2})\) could be represented by

\[ \sigma_{eq} = 946(\sigma_{eq} + 0.003) \quad \text{for} \quad 0 < \sigma_{eq} < 0.034 \]  

\[ \sigma_{eq} = 482(\sigma_{eq} - 0.0105) \quad \text{for} \quad \sigma_{eq} > 0.034 \]  

where \( \sigma_{eq} \) is the equivalent stress in \( \text{MN m}^{-2} \), \( \sigma_{eq} \) is the equivalent strain, \( \nu \) is Poisson's ratio, and \( E \) is Young's modulus. The best fitting of the data in Fig. 1 to the \( Y_B-R \) relationship in equation (1) gives \( R_c = 0.442 \).

Figure 2 shows Shima's measured relative densities of sintered copper specimens with different initial relative densities as a function of height strain in frictionless upsetting, along with those obtained by analytical and
numerical methods. Before discussing the results, the analytical method is first explained. Assuming homogeneous and rigid plastic deformation in simple upsetting and the following relationship (equation (19)) between Poisson's ratio $v$ and relative density $R$, suggested by Zhidanovich,\textsuperscript{13} Kim and Cho\textsuperscript{14} derived a relationship between relative densities before and after deformation (equation (20))

$$ v = 0.5R^n $$

$$ R = \frac{\exp(n\epsilon)}{\exp(n\epsilon) + (1 - R^n)/R^n} $$

where $\epsilon$ is the height strain, defined as $-\ln(h/h_0)$, with $h_0$ and $h$ being the height of the specimen before and after deformation respectively. Kuhn\textsuperscript{15} obtained $n = 2$ in equation (19) from specimens on porous metals. In fact, the relationship $v = 0.5R^2$ was also used to derive the yield condition of equation (1). The numerical method used is the elastoplastic finite element method described above. Figure 3 shows the initial mesh for the finite element calculation. The numerical method and equation (20) with $n = 2$ are seen to give almost identical results, implying that the numerical method is very accurate. The good correlation between the measured and calculated results indicates that the finite element method based on equation (1) is reliable.

Figure 4 shows the radial strain of the porous copper specimens as a function of height strain during frictionless upsetting. The values calculated by the finite element method are in very good agreement with the data of Shima and Oyane.\textsuperscript{9}

Mori et al.\textsuperscript{4} performed upsetting experiments on sintered cylindrical copper specimens under sticking and Coulomb frictions. The initial height and diameter of the specimens were both 20 mm. The mechanical properties ($\sigma_{eq}$ in
Calculated and measured properties of 20% reduced sintered copper specimens under sticking friction

MN m⁻²) of the non-porous copper specimens (ν = 0.3, E = 117 GN m⁻², σ₀ = 94 MN m⁻²) were as follows:\(^4\)

\[ \sigma_{eq} = 431(e_{eq} + 0.00851)^{0.32} \]

Since \( R_c \) of this system is not known, \( R_c \) for the previous example, \( R_c = 0.442 \), was used in the calculation.

Calculations have been performed for various friction conditions on material with an initial relative density of 0.81. Figure 5a-d shows the calculated grid distortion, relative density, hydrostatic pressure, and the measured relative density (based on hardness) distributions\(^4\) in a specimen reduced by 20% assuming a friction coefficient of 0.25. Figure 6a-d shows the results for sticking friction. It can be seen that the calculated relative density distributions are comparable with the measured data. It is seen from Figs. 5 and 6 that for 20% reduction, the relative density is low half way down the side surface and the centre of the die/metal interface. The grid distortion and relative density distributions in 40 and 60% reduced specimens calculated assuming a friction coefficient of 0.25 are shown in Fig. 7. The highest density is observed at the centre of the workpiece and near the edge region of the die/metal interface. It can be seen that folding occurs at the outer edge region of the 40% reduced specimen. Most regions reach relative densities above 0.99 and the density is lowest at a point half way down the side surface where the specimen becomes barrelled at a reduction of 60%. Figure 8 shows the measured load-displacement data\(^4\) along with values calculated for various friction coefficients. It can be seen that the forming loads are insensitive to friction and the calculated values are in good agreement with the measured data.

Figure 9 shows the average relative density of a sintered porous alloy steel with an initial relative density of 0.825.

Calculated grid distortion and density distributions in 40% reduced (top) and 60% reduced (bottom) sintered copper specimens assuming a friction coefficient of 0.25.

\[ A = 0.850E + 00 \]
\[ B = 0.878E + 00 \]
\[ C = 0.906E + 00 \]
\[ D = 0.934E + 00 \]
\[ E = 0.962E + 00 \]
\[ F = 0.990E + 00 \]
8 Upsetting loads as function of height strain for various friction conditions: Mori's experimental results\textsuperscript{4} (symbols) are compared with calculated values for given coefficient of friction

as a function of height strain measured by Kim and Cho.\textsuperscript{14} They analysed the experimental data based on equation (20). The best fitting of the data yielded \( n = 2.37 \) rather than \( n = 2 \) in this equation which was derived assuming frictionless homogeneous deformation. However, finite element analysis of the deformation assuming a friction coefficient of 0.1 gives a better result as shown in Fig. 9. The calculated effect of coefficient of friction on average relative density versus height strain is shown in Fig. 10. The average relative density increases slowly with increasing friction. It follows from the results in Figs. 9 and 10 that it is not necessary to take values other than 2 as the \( n \) value.

Indenting
Tabata et al.\textsuperscript{16} measured the Brinell hardness of sintered copper specimens as a function of \( d/D \), where \( d \) and \( D \) are the respective diameters of the indented mark and the indenter, and the results are shown in Fig. 11. Hardness increased monotonically with increasing \( d/D \), except for specimens of higher initial densities. It is to be expected that the hardness increases with increasing initial relative density. There had, however, previously been no theoretical explanation of the hardness data. The authors of the present paper have made a finite element analysis of the two-dimensional axisymmetric frictionless indentation of specimens 24.5 mm in diameter and 30 mm in height by a 5 mm diameter hemispherical iron indenter. The dimensions of the specimens and indenter are identical to those used in Tabata's experiment. The mechanical properties (\( \sigma_{eq} \) in MN m\textsuperscript{-2}) of the non-porous copper specimens (\( v = 0.3 \), \( E = 117 \) GN m\textsuperscript{-2}, \( \sigma_0 = 54 \) MN m\textsuperscript{-2}) and \( R_c \) show the following relationships\textsuperscript{16,17}

\[
\sigma_{eq} = 558v^{0.47}, \ R_c = 0.375
\]

The initial mesh for indentation is shown in Fig. 12. The Brinell hardness (HB) is load \( F \) divided by indented surface area\textsuperscript{18}

\[
HB = F/[(\pi D/2)[D-(D^2-d^2)]^{1/2}] = F/\pi D t \quad \text{.} (21)
\]

where \( F, D, d, \) and \( t \) are applied load, diameter of indenter, diameter of indentation, and depth of indentation respectively. The calculated Brinell hardness as a function of \( d/D \) are shown in Fig. 11 along with the Brinell hardnesses
measured by Tabata et al. A good correlation between the measured and calculated values can be seen. The Brinell hardness is usually measured at constant $F$ and $D$. For an indenting load of 2450 N, the usual Brinell hardness value is shown in Fig. 13 as a function of initial relative density. A linear relationship between the initial relative density and the hardness is seen within the experimental range of Tabata et al.

Calculated grid distortions and relative density distributions in the specimens with initial relative densities of 0.8 and 0.9 during indentation are shown in Fig. 14. The shape of the densification zone under the indenter is similar to that of the plastic zone of non-porous metal indented by a spherical indenter. It is interesting to note that the region next to the indenter sinks rather than bulges.

**CONCLUSIONS**

The calculated radial strain versus height strain relationships and relative densities of porous metals subjected to frictionless upsetting are in good agreement with the experimental values. The calculated density distributions in porous specimens upset under various friction conditions are comparable with the measured distributions. The average relative density in upset porous metals increases slowly with increasing friction. The Brinell hardness calculated for a given load and indenter diameter increases linearly with the initial relative density of the specimen and correlates well with the measured data, although the measured hardness tends to increase with increasing ratio of indentation to indenter diameter.

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**REFERENCES**

First Announcement

SUPERPLASTICITY – 60 YEARS AFTER PEARSON

7–8 December 1994
Manchester

Organised by the Superplastic Forming Committee of the Manufacturing Technology Division of
The Institute of Materials

The meeting will be held to commemorate the 60th anniversary of the publication by the former Institute of Metals of the classic paper by C. E. Pearson entitled 'The viscous properties of extruded eutectic alloys'. Not only did Pearson observe very high tensile strains in metallic materials but he also understood important microstructural aspects of the phenomenon, which subsequently became known as superplasticity. He was also the first to demonstrate the feasibility of superplastic biaxial bulge forming using gas pressure, a procedure which is the basis of the forming techniques employed commercially today.

Scope
The conference will review the current understanding of superplastic behaviour in metals and other crystalline solids, but in particular will be concerned with the current status of, and future trends in, superplastic forming of commercially useful shapes. The markets in which superplastically formed components are finding applications will be reviewed as will the progress in forming machinery, associated equipment, and manufacturing techniques.

The conference is intended to include both invited and contributed papers dealing with mechanisms of superplastic flow, cavitation, diffusion bonding/superplastic forming, superplasticity in ceramics and intermetallics, and high strain rate superplasticity. In addition contributed papers are intended to deal with Al alloys, Ti alloys, stainless steels, and commercial aspects of diffusion bonding and/or superplastic forming.

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