Symmetry and Anisotropy

Structure, Properties, Sample and Material, Texture and Anisotropy, Symmetry…
Objectives

<Symmetry>

• Symmetry Operators & Matrix representation.
• Effect of crystal and sample symmetry on the Euler space for unique representation of orientations.
• In Cubic crystal - Orthorhombic sample symmetry, the 3 equivalent points in $\pi/2 \times \pi/2 \times \pi/2$ “box” or “reduced space”.
• Concept of “Fundamental Zone”.
• Fiber Symmetry & Orthotropy
• Typical Fiber(full & Partial) in Metals

<Anisotropy>

• Anisotropy related with Orientation & Symmetry
• Various Physical Properties depending on Orientation & Texture
Introduction

- **Anisotropy**
  - “How the sample is turned (Oriented)?”
  - Property dependency on the Orientation
  - Due to the its *Structure (External & Internal)*

- **Structural Elements Responsible for Anisotropy**
  - Shape: Geometrical Orientation
  - Orientation.: Crystallographic Orientation
  - Texture as a Totality of Crystallographic Orientation
State & Properties

- Material state & Properties are determined by:
  - Current state
  - Microscopic view
  - Crystal structure
  - Defect structure
  - Microstructure $\rightarrow$ Texture
  - Process History
  - Measuring Environment
  - $\ldots$, And so on
Texture
- Element of Microstructure
- Non-random Distribution of Grain Orientation
- Morphology of Grains

Property
- Relation between Stimulus and Response
  \[ \text{Response} = \text{Modulus} \times \text{Stimulus} \]
  \[ \rightarrow \text{Modulus} = M(m_{ijkl}, g) \]
  \[ m_{ijkl} : \text{Modulus of Single Crystal} \]
  \[ g : \text{Grain Orientation} \]
**Example: Muscovite**

Heterogeneous Structure  
Lattice Structure

Schematic Anisotropy  
Relevant Property

*Anisotropy due to Geometry & Orientation* $\rightarrow$ *Property Anisotropy with Orientation*  
$\rightarrow$ *Quarter Section of Orientation is enough to describe Anisotropy*  
*Due to Symmetry of Microstructure (Orientation)*

**Anisotropy and Symmetry**
Symmetry
Effect of Symmetry

a) 3-fold, b) 4-fold, and c) 6-fold rotational symmetry.
Why Symmetry Required?

Quantification of Anisotropy of Property

- Degree of Anisotropy
  - Ratio of Max/Min: Simplest
  - Tensor Description: Fairly Complex function
- Physical Parameters
  - More fundamental Approach Requires Distribution Function in G-Space
  - For Mathematical Convenience, Symmetry Consideration is Necessary
- Symmetry Characteristics
  - Symmetry Characteristics do not affect its anisotropy.
  - Centrosymmetry (center of inversion)
Summary of Projection System

Cubic Unit Cell

- RD
- TD
- ND

- : Origin
- : Projection Origin
- : Crystal
- : Sample

Anisotropy and Symmetry
Orientations & Lattice

- Two Meanings of the term ‘Orientation’
  - Orthogonal Translation
  - Rotation ➔ Definition in Texture Analysis

Same Lattice orientation:
- all the unit cell are aligned the same way.

Laue Orientation
- obtained by X-ray diffraction experiment.
- the term ‘orientation’ used in General
- Crystal Lattice Symmetry should be considered
**Crystal Symmetries**

- **Crystal structure**
  - Basic Structural Element
  - Atomic Structural Symmetry is contained.

- **Crystal Classes and Laue groups**
  - Unit cells for 7 lattice types

- **Crystal Symmetry**

- **Anisotropy**

- **Symmetry**
### Crystal Symmetries for Orientation

<table>
<thead>
<tr>
<th>CRYSTAL SYSTEM</th>
<th>crystal class</th>
<th>no. of sym. el.</th>
<th>no. of centro-sym. tensor prop.’s.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Laue</td>
<td>rot.</td>
<td>sym. 2nd-rank</td>
</tr>
<tr>
<td>triclinic</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>monoclinic</td>
<td>2/m</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>orthorhombic</td>
<td>mmm</td>
<td>222</td>
<td>3</td>
</tr>
<tr>
<td>tetragonal</td>
<td>4/m</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4/mmm</td>
<td>422</td>
<td>8</td>
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<tr>
<td>trigonal</td>
<td>3</td>
<td>3</td>
<td>2</td>
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<td></td>
<td>3m</td>
<td>3</td>
<td>6</td>
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<tr>
<td></td>
<td>6/m</td>
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<td>2</td>
</tr>
<tr>
<td>hexagonal</td>
<td>6/mmm</td>
<td>622</td>
<td>12</td>
</tr>
<tr>
<td>cubic</td>
<td>m3</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>m3m</td>
<td>432</td>
<td>24</td>
</tr>
</tbody>
</table>

† : Subgroup
11 Laue Groups

- Inversion center addition
- 3 Branches
  - Tetragonal Branch
  - Hexagonal Branch
  - Cubic Branch

Anisotropy and Symmetry
Cubic Branch

▶ The presence of *four* triad symmetry elements in all these groups on <111>.

▶ Cubic Crystal mostly fall under $m3m$. 

Groups: mathematical concept, very useful for symmetry
Rotational Symmetry Operators

- All the possible rotation elements are easily enumerated.
- Applying an inversion center:
  - Laue group is obtained.
- All rotation elements, grouped into three branches.
  - Cubic branch
  - Tetragonal branch
  - Hexagonal branch

<table>
<thead>
<tr>
<th>tetragonal branch</th>
<th>hexagonal branch</th>
<th>cubic branch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

Table II. Symmetry operators of rotation groups

The dashed boxes in this column make up group 4.
The dashed boxes in this column make up group 32.
The dashed box in this column comprises the 3-fold axes only.

Anisotropy and Symmetry
Sample Symmetry

**Triclinic**: Extremely Aligned

**Monoclinic**, 2 : Torsion, shear

**Orthorhombic**, *mmm* : Rolling, plane strain compression

**Axisymmetric**: $C_\infty$

$\rightarrow$ Fiber Symmetry

**Isotropic**: Randomly Oriented

*Randomness Increase*

Anisotropy and Symmetry
**Graphical Representation**

**Rotation Axis**

3 fold  4 fold  6 fold  Axi-Symmetry; $C_\infty$

Triclinic  Monoclinic  Orthorhombic  Random

**Mirror Plane**

Anisotropy and Symmetry
Sample Symmetry Example

One-directional elongation

1) All grains elongated in the same sample direction.
2) Some crystal axes are aligned. : Texture
3) Need not coincide the preferred axes for crystallographic and morphological features

Example

- In-situ Tensile Micro-Deformation of ECAP
- EBSD Measurement at Same Location
Texture Evolution during ECAP by In-situ Microdeformation

- Deformation mode?
- Orientation rotation?

1. Tensile deformation
2. Shear deformation
3. Complex deformation

Anisotropy and Symmetry
In-situ Deformation System

- FEG-SEM (JSM 6500F)
- EBSD (Oxford INCA Crystal)

Pure Al single crystal (t = 1mm)
1050 Al polycrystal (t = 1 mm)

Deformation Stage

200 µm
200 µm

Infrared Camera

Anisotropy and Symmetry
Morphological Change of Surface in single crystal

<table>
<thead>
<tr>
<th>Initial orientation</th>
<th>Slip system</th>
<th>Schmid factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(546) [273]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slp system</th>
<th>Schmid factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(111) [T10]</td>
<td>0.119</td>
</tr>
<tr>
<td>(111) [10T]</td>
<td>0.013</td>
</tr>
<tr>
<td>(111) [01T]</td>
<td>0.132</td>
</tr>
<tr>
<td>(1T1) [0T1]</td>
<td>0.316</td>
</tr>
<tr>
<td>(1T1) [101]</td>
<td>0.079</td>
</tr>
<tr>
<td>(1T1) [T0]</td>
<td>0.395</td>
</tr>
<tr>
<td>(11T) [011]</td>
<td>0.210</td>
</tr>
<tr>
<td>(11T) [T0T]</td>
<td>0.265</td>
</tr>
<tr>
<td>(1T1) [T10]</td>
<td>0.478</td>
</tr>
<tr>
<td>(1[11]) [01]</td>
<td>0.416</td>
</tr>
<tr>
<td>(1T1) [10]</td>
<td>0.208</td>
</tr>
</tbody>
</table>

Slip System Symmetry → Triclinic

Anisotropy and Symmetry
### Orientation Rotation (RD)

<table>
<thead>
<tr>
<th></th>
<th>No-Strain</th>
<th>2.6% Strain</th>
<th>6.2% Strain</th>
<th>10.2% Strain</th>
<th>13.4% Strain</th>
<th>36.8% Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

**Directionally Aligned with Deformation: Orthorhombic**

**Anisotropy and Symmetry**
Summary of In-situ Tensile Test

- In-situ Tensile Micro-Deformation of ECAP
- EBSD Measurement at Same Location

→ What We Found?

1. Each Grain Deformation is Triclinic Symmetry
2. Deformation Axis is Not Coincident with Sample Axis
3. Apparently Orthorhombic Symmetry due to Statistical Average

To Obtain Statistically Reliable Data, How Big Sample?
Sample Size

10 × 10 × 10 = 1000
8 × 8 × 8 = 512

Large enough to have a small fraction of its grains in non-representative locations.
**Symmetry & Euler Angle**

- Crystal Symmetry Operates in a frame of Crystal axes.
  - Crystal Symmetry affects the second & third angles.

- Sample Symmetry Operates in a frame of Sample axes.
  - Sample Symmetry affects the first & second angles.

- Combination of Crystal & Sample Symmetry:
  - *Crystal-Sample Symmetry*
  - e.g. *Cubic-Orthorhombic*, or *Hexagonal-Triclinic*.
Fundamental Zone

- The fundamental zone: the subset of Orientation Space within which each orientation is described by a single, unique point.
  - Minimum Orientation space to describe all orientations.
  - e.g: Standard stereographic triangle (SST) in Cubic Crystals.

- The size of the fundamental zone depends on the amount of symmetry present in both Crystal and Sample space.
  - More symmetry ⇒ Smaller Euler fundamental zone.

- Note that the 90x90x90° region typically used for cubic [crystal]+orthorhombic [sample] symmetry is not a fundamental zone because it contains 3 copies of the actual zone!
Anisotropy and Symmetry

Meaning of Euler angles

- First two angles, $\phi_1$ and $\Phi$, the position of the [001] crystal direction relative to the specimen axes.
- Think of rotating the crystal about the ND (1st angle, $\phi_1$); then rotate the crystal out of the plane (about the [100] axis, $\Phi$);
- 3rd angle ($\phi_2$) tells Rotation of the crystal about [001].
Sample Symmetry Element
e.g. diad on ND
(associated with $\phi_1$)

Crystal Symmetry Element
e.g. rotation on [001]
(associated with $\phi_2$)
Choice of Section Size

- Quad, Diad symmetry elements are easy to incorporate, but Triads are highly inconvenient.
- Four-fold rotation elements (and mirrors in the orthorhombic group) are used to limit the third, $\phi_2$, (first, $\phi_1$) angle range to 0–90°.
- Second angle, $\Phi$, has range 0–90° (diffraction adds a center of symmetry).
Section Sizes: Crystal - Sample

▶ Cubic–Orthorhombic:
  \(0 \leq \phi_1 \leq 90^\circ, 0 \leq \Phi \leq 90^\circ, 0 \leq \phi_2 \leq 90^\circ\)

▶ Cubic–Monoclinic:
  \(0 \leq \phi_1 \leq 180^\circ, 0 \leq \Phi \leq 90^\circ, 0 \leq \phi_2 \leq 90^\circ\)

▶ Cubic–Triclinic:
  \(0 \leq \phi_1 \leq 360^\circ, 0 \leq \Phi \leq 90^\circ, 0 \leq \phi_2 \leq 90^\circ\)

▶ But, these limits do not delineate a fundamental zone.
Points related by triad symmetry element on <111> (Triclinic sample symmetry)

Take a point, e.g. “B”; operate on it with the 3-fold rotation axis (blue triad); the set of points related by the triad are B, B’, B”, with B”” being the same point as B.
Example of 3-fold symmetry

S Component : \( \{123\} <634> \)

3 Related Locations

\( S_I : \{59, 37, 63\} \)
\( S_{II} : \{27, 58, 18\} \)
\( S_{III} : \{53, 74, 34\} \)

Regions I, II & III are related by the triad symmetry element, i.e. 120° about <111>

10° scatter about component.
Anisotropy and Symmetry

Effect of 3-fold axis

Section in $\phi_1$ cuts through more than one Subspace

$\cos \Phi = \frac{\cos \Phi_2}{\sqrt{1 + \cos^2 \Phi_2}}$
This table summarizes the differences between the two standard data sets found in popLA Orientation Distribution files. A `name.SOD` contains exactly the same data as `name.COD` - the only difference is the way in which the OD space has been sectioned.

<table>
<thead>
<tr>
<th>Crystallite Orientation Distribution</th>
<th>Sample Orientation Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>COD</td>
<td>SOD</td>
</tr>
<tr>
<td>fixed third angle in each section</td>
<td>fixed first angle in each section</td>
</tr>
<tr>
<td>sections in $(\phi_1,\Phi)/(\Psi,\Theta)$</td>
<td>sections in $(\phi_2,\Phi)/(\phi,\Theta)$</td>
</tr>
<tr>
<td>$\phi_2/\phi = \text{constant}$</td>
<td>$\phi_1/\Psi = \text{constant}$</td>
</tr>
<tr>
<td>Reference = Sample Frame</td>
<td>Reference = Crystal Frame</td>
</tr>
<tr>
<td>Average of sections-&gt; (001) Pole Figure</td>
<td>Average of sections-&gt; ND Inverse Pole Figure</td>
</tr>
</tbody>
</table>
Rotations: definitions

- Rotational Symmetry Elements exist whenever you can rotate a physical object and result is → Indistinguishable from Starting Orientation
- Rotations can be expressed in a Simple Matrices → Orientation Matrix
- Rotations is a transformations of first kind → Determinant of Matrix = +1

Note: Second kind; determinant = -1
Anisotropy and Symmetry

The rotation can be converted to a matrix (passive rotation) where $\delta$ is the Kronecker delta and $\varepsilon$ is the permutation tensor.

### Rotations (Active): Axis-Angle Pair

A rotation is commonly written as $(\vec{r}, \theta)$ or as $(\vec{n}, \omega)$. The figure illustrates the effect of a rotation about an arbitrary axis, $OQ$ (equivalent to $\vec{r}$ and $\vec{n}$) through an angle $\alpha$ (equivalent to $\theta$ and $\omega$).

\[
g_{ij} = \delta_{ij} \cos \theta + r_i r_j (1 - \cos \theta) + \sum_{k=1,3} \varepsilon_{ijk} r_k \sin \theta
\]

(This is an active rotation: a passive rotation = axis transformation)

Compare with active rotation matrix!
Rotation Matrix from Axis-Angle Pair

\[ g_{ij} = \delta_{ij} \cos \theta + r_ir_j(1 - \cos \theta) \]
\[ + \sum_{k=1,3} \varepsilon_{ijk} r_k \sin \theta \]
\[ = \begin{pmatrix}
\cos \theta + u^2(1 - \cos \theta) & uv(1 - \cos \theta) + w\sin \theta & uv(1 - \cos \theta) - v\sin \theta \\
uv(1 - \cos \theta) - w\sin \theta & \cos \theta + v^2(1 - \cos \theta) & vw(1 - \cos \theta) + u\sin \theta \\
uv(1 - \cos \theta) + v\sin \theta & vw(1 - \cos \theta) - u\sin \theta & \cos \theta + w^2(1 - \cos \theta)
\end{pmatrix} \]
Rotation Matrix examples

• Diad on z: \([uvw] = [001]\), \(\theta = 180^\circ\) - substitute the values of \(uvw\) and angle into the formula

\[
g_{ij} = \begin{pmatrix}
\cos 180 + 0^2(1-\cos 180) & 0*0(1-\cos 180)+1*\sin 180 & 0*1(1-\cos 180)-0\sin 180 \\
0*0(1-\cos 180)-w\sin 180 & \cos 180+0^2(1-\cos 180) & 0*1(1-\cos 180)+0\sin 180 \\
0*1(1-\cos 180)+0\sin 180 & 0*1(1-\cos 180)-0\sin 180 & \cos 180+1^2(1-\cos 180)
\end{pmatrix} = \begin{pmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

• 4-fold on x: \([uvw] = [100]\) \(\theta = 90^\circ\)

\[
g_{ij} = \begin{pmatrix}
\cos 90+1^2(1-\cos 90) & 1*0(1-\cos 90)+w\sin 90 & 0*1(1-\cos 90)-0\sin 90 \\
0*1(1-\cos 90)-0\sin 90 & \cos 90+0^2(1-\cos 90) & 1*0(1-\cos 90)+1\sin 90 \\
0*1(1-\cos 90)+0\sin 90 & 0*0(1-\cos 90)-1\sin 90 & \cos 90+0^2(1-\cos 90)
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{pmatrix}
\]
Matrix representation of rotation point groups

What is a group? A group is a set of objects that form a closed set: if you combine any two of them together, the result is simply a different member of that same group of objects. Rotations in a given point group form closed sets - try it for yourself!

Note: the 3rd matrix in the 1st column (x-diad) is missing a “-” on the 33 element - corrected in this slide.

Anisotropy and Symmetry
Other Symmetry Operators

- Symmetry Operators of the *second kind*: Inversion center & mirrors; determinant $= -1$
- Inversion simply reverses any vector $(x, y, z) \rightarrow (-x, -y, -z)$
- Mirrors operate through a mirror axis.
  - An x-mirror is a mirror in the plane $x=0$ as $(x, y, z) \rightarrow (-x, y, z)$
Examples of Symmetry Operators

\[
\begin{align*}
&\begin{pmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix} & & \begin{pmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} & & \begin{pmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{pmatrix} \\
\text{Diad on } z: & & \text{Mirror on } x: & & \text{Inversion Center:}
\end{align*}
\]

(1st kind) \hspace{2cm} (2nd kind) \hspace{2cm} (2nd kind)
How to Use a Symmetry Operator?

- Convert Orientation to a matrix.
  Miller Indices, Euler Angle, RF Vector → Matrix
- Matrix Multiplication with the symmetry operator and the orientation matrix.
- Convert Matrix back to Orientation
- The two sets of Orientation Represent Indistinguishable objects. (for crystal symmetry)
Example

Goss: \(\{110\}\langle001\rangle\):  
\[
\begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
1 & 0 & 0
\end{pmatrix}
\]

Pre-multiply by z-diad:  
\[
\begin{pmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
1 & 0 & 0
\end{pmatrix}
\]

which is \(\{-1-10\}\langle001\rangle\)

Anisotropy and Symmetry
**Order of Matrices**

- Order depends on whether *Crystal* or *Sample* symmetry elements are applied.
- For Crystal Symmetry Operator, $O_{xtal}$,
  \[ \rightarrow \text{Pre-multiplication to Orientation Matrix.} \]
- For Sample Symmetry Operator, $O_{sample}$,
  \[ \rightarrow \text{Post-multiplication to Orientation Matrix} \]

Note: Orientation

Sample Axis \[ \rightarrow \] Crystal Axis

\[ g = (O_{sample}) \quad g \quad (O_{xtal}) \]
Full Symmetry Relationships

\[ g' = O_{\text{crystal}} g O_{\text{sample}} \]

- Note that the result of applying any available operator is equivalent to from the starting configuration
  \[ \Rightarrow \text{Physically Indistinguishable in Crystal Symmetry} \]
- In a sample symmetry operator, the result is generally physically different from the starting position.
  \[ \Rightarrow \text{Because Sample Symmetry is a statistical symmetry, not an exact physical symmetry} \]
Anisotropy and Symmetry

How many Equivalent Points?

- Each Symmetry Operator relates a pair of points in orientation (Euler) space.

- Each operator divides the available space by a factor of the order of the rotation axis.
  
  If there are four symmetry operators in the group, then the size of orientation space is decreased by four.

- This suggests that the orientation space is smaller than the general space by a factor equal to the number of general poles.

Anisotropy and Symmetry
Crystal Symmetry

Sections at constant values of the third angle

<table>
<thead>
<tr>
<th>Laue group</th>
<th>Section range</th>
<th>COD</th>
<th>Oblique sections</th>
<th>Multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic</td>
<td>m3m</td>
<td>90°</td>
<td>45°</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>m3</td>
<td>180°</td>
<td>90°</td>
<td>3</td>
</tr>
<tr>
<td>Tetragonal</td>
<td>4/mmm</td>
<td>90°</td>
<td>45°</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4/m</td>
<td>180°</td>
<td>90°</td>
<td>1</td>
</tr>
<tr>
<td>Orthorhombic</td>
<td>mmm</td>
<td>180°</td>
<td>90°</td>
<td>1</td>
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<tr>
<td>Hexagonal</td>
<td>6/mmm</td>
<td>60°</td>
<td>30°</td>
<td>1</td>
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<tr>
<td></td>
<td>6/m</td>
<td>120°</td>
<td>60°</td>
<td>1</td>
</tr>
<tr>
<td>Trigonal</td>
<td>3/m</td>
<td>120°</td>
<td>60°</td>
<td>1</td>
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<td></td>
<td>3</td>
<td>240°</td>
<td>120°</td>
<td>1</td>
</tr>
<tr>
<td>Monoclinic</td>
<td>2/m</td>
<td>360°</td>
<td>180°</td>
<td>1</td>
</tr>
<tr>
<td>Triclinic</td>
<td>1</td>
<td>—</td>
<td>360°</td>
<td>1</td>
</tr>
</tbody>
</table>

\[^{*}\text{It is sometimes advantageous to use a semi-circle and cut the section range in half.}\]
Sample Symmetry

Sections at constant values of the first angle

Anisotropy and Symmetry
Cubic symmetry

- $O(432)$ has 24 operators (i.e. order=24)
  $O(222)$ has 4 operators (i.e. order=4)

→ Why not divide the volume of Euler space ($8 \pi^2$, or, $360° \times 180° \times 360°$) by $24 \times 4 = 96$ to get $\pi^2/12$ (or, $90° \times 30° \times 90°$)?

→ We leave out a triad axis, so divide by $8 \times 4 = 32$ to get $\pi^2/4$ ($90° \times 90° \times 90°$).
Orthotropy

- Idealized Sheet Rolling
- Plane Strain Compression
- Orthorhombic crystal + inversion center (class mmm); *orthotropy*
Symmetry Operation

• **Crystal Symmetry:**
  A Combination of 4- and 2-fold crystal axes (2x4=8 elements) reduce the range of $\Phi$ from $\pi$ to $\pi/2$, and $\phi_2$ from $2\pi$ to $\pi/2$.

• **Sample symmetry:**
  2-fold sample axes (4 elements in the group) reduce the range of $\phi_1$ from $2\pi$ to $\pi/2$.

• **Volume:** $\{0 \leq \phi_1, \Phi, \phi_2 \leq \pi/2\} \rightarrow \pi^2/4$
Cubic Crystal Symmetry in G-Space

\[ H' = \{0 \leq \varphi_1 \leq 2\pi, \ 0 \leq \phi \leq \pi/2, \ 0 \leq \varphi_2 \leq \pi/2\} \]

4- & 2-fold crystal axes (2x4=8 elements)

→ Range of \(\Phi\) from \(\pi\) to \(\pi/2\), & \(\phi_2\) from \(2\pi\) to \(\pi/2\)
Orthorhombic Sample Symmetry in G-Space

2-fold sample axes (4 elements in the group)

→ Range of \( \phi_1 \) from \( 2\pi \) to \( \pi/2 \)

Anisotropy and Symmetry
Cubic-Orthorhombic Symmetry

Why \( \{0 \leq \phi_1, \Phi, \phi_2 \leq \pi/2\} \) ?

- \( O(432) \) has 24 operators (i.e. order=24)
- \( O(222) \) has 4 operators (i.e. order= 4)

\( \Rightarrow \) Why Not Divide the volume of Euler space from \((8\pi^2, \text{or}, 360^\circ\times180^\circ\times360^\circ)\) by \(24\times4=96\)
  to \( (\pi^2/12 \text{ (or, } 90^\circ\times30^\circ\times90^\circ) \)?

- Answer: Leave Out a Triad axis : \( O(4\overline{3}2) \)
  \( \Rightarrow \) Because of the Awkward Shapes (Triangular Prism)
  \( \Rightarrow \) So Divide by \(8\times4=32\) to get \(\pi^2/4 (90^\circ\times90^\circ\times90^\circ)\).
How many Equivalent Points?

For Cubic-Orthorhombic Crystal+Sample Symmetry, we use a range $90^\circ \times 90^\circ \times 90^\circ$ for the three angles, giving a volume of $90^\circ^3$ (or $\frac{\pi^2}{4}$ in radians).

In this (reduced) space there are 3 equivalent points for each orientation (texture component). Both sample and crystal symmetries must be combined together to find these sets.

Fewer (e.g. Copper) or more (e.g. cube) equivalent points for each component are found if the the component coincides with one of the symmetry elements.
Crystal Symmetry Relationships (432) in Euler Space

3-fold axis

\[ \phi_2 = 0^\circ \quad 90^\circ \quad 180^\circ \quad 270^\circ \quad 360^\circ \]

\[ \Phi = 180^\circ \]

Note: points related by triad (3-fold) have different \( \phi_1 \) values.

Anisotropy and Symmetry
Orthorhombic Sample Symmetry (mmm) Relationships in Euler Space

$\phi_1 = 0^\circ$  

90° **diad** 180° 270° 360°

$\Phi$

$\Phi = 180^\circ$

2-fold screw axis changes $\phi_2$ by $\pi$
Crystal Symmetry (432) to (231)[3-46] S component

\[ \phi_1 \text{ values noted} \]

\[ \phi_2 \]

Anisotropy and Symmetry
Regions I, II and III are related by the triad symmetry element, i.e. 120° about <111>.

**S component in $\phi_2$ sections**

<table>
<thead>
<tr>
<th>$\phi_2$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_2$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>$\Phi=45^\circ$</td>
<td>5°</td>
<td>$\Phi=54.7^\circ$</td>
<td>10°</td>
</tr>
<tr>
<td>20°</td>
<td>$\Phi=45^\circ$</td>
<td>25°</td>
<td>$\Phi=50^\circ$</td>
<td>30°</td>
</tr>
<tr>
<td>40°</td>
<td>$\Phi=54.7^\circ$</td>
<td>45°</td>
<td>$\Phi=55^\circ$</td>
<td>50°</td>
</tr>
<tr>
<td>60°</td>
<td>$\Phi=54.7^\circ$</td>
<td>65°</td>
<td>$\Phi=70^\circ$</td>
<td>70°</td>
</tr>
<tr>
<td>80°</td>
<td>$\Phi=45^\circ$</td>
<td>85°</td>
<td>$\Phi=75^\circ$</td>
<td>90°</td>
</tr>
<tr>
<td>$\phi_2$=const</td>
<td>$\phi_2$=const</td>
<td>$\phi_2$=const</td>
<td>$\phi_2$=const</td>
<td>$\phi_2$=const</td>
</tr>
</tbody>
</table>

**Anisotropy and Symmetry**
Special Points

Copper : 2
Brass  : 3
S      : 3
Goss   : 3
Cube   : 8
Dillamore : 2
Sample Symmetry Relationships in Euler Space: special points

$\phi_1 = 0^\circ$  $90^\circ$  $180^\circ$  $270^\circ$  $360^\circ$

Cube lies on the corners

Copper, Brass, Goss lie on an edge

Anisotropy and Symmetry
Special Points: Explanations

• Points Coincident with Symmetry axes may also have equivalent points, often on the edge.

• Cube should be a single point but each corner is equivalent and visible.

• Goss, Brass: a single point becomes 3 because on $\phi_2=0$ plane

• Copper & Dillamore: 2 points because one point in interior & another on face
**3D Views**

a) Brass   b) Copper   c) S
d) Goss   e) Cube   f) combined texture

1: \{35, 45, 90\}, Brass,
2: \{55, 90, 45\}, Brass
3: \{90, 35, 45\}, Copper,
4: \{39, 66, 27\}, Copper
5: \{59, 37, 63\}, S,
6: \{27, 58, 18\}, S,
7: \{53, 75, 34\}, S
8: \{90, 90, 45\}, Goss
9: \{0, 0, 0\}, cube*
10: \{45, 0, 0\}, rotated cube

Anisotropy and Symmetry
Fiber Symmetry

Anisotropy and Symmetry
**Fiber Symmetry of Wire**

- An Initially Isotropic Wire
  - Pulled in tension or in Compression
- Properties of the Resulting Sample
  - Transversely isotropic
  - The Shape of Sample is not Considered Part
- Drawn wire
  - Texture Gradient along Radial Direction
- A Uniform Intensity Line in orientation space
  - Identified as a fiber.
Texture of Gold Bonding Wire

(100) Texture

(111) Texture

Anisotropy and Symmetry
Pole Figure for Wire Texture

<100> // DD : (111) Pole

Rotation Around DD
Anisotropy and Symmetry

Pole Figure for Wire Texture

{111}

In this case, <100> // F.A.
(Partial) Fibers

- A Uniform Intensity Line in orientation space → Identified as a fiber.
- Most deformation texture occurs along fibers.
- Caution required in defining fibers: location of maximum density often not on ideal fiber as defined in textbooks.
**α fiber in rolled fcc metals**

- α fiber in rolled fcc metals: \{110\} ND fiber
- Line from Goss \{110\}<001> to Brass \{110\}<1-12>
- 3 branches of α fiber exist because of the triad symmetry element.
- Typically examination in only one branch.
β fiber in rolled fcc metals

- β fiber in rolled fcc metals
  - Line from Copper {112}<11-1>
    - to Brass {110}<1-12>
- 3 branches of β fiber exist
  - because of the triad symmetry element.
- Typically examine only 45°<Φ<90° branch.
Fiber Plots: Intensity along $\beta$ fiber
### Fibers in rolled bcc metals

<table>
<thead>
<tr>
<th>Designation</th>
<th>Orientation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$, or $\langle 110 \rangle$</td>
<td>${001} \langle 110 \rangle$ to ${111} \langle 110 \rangle$</td>
<td>$&lt;110&gt; \parallel$ RD; prominent in bcc rolling textures</td>
</tr>
<tr>
<td>$\gamma$, or ${111}$</td>
<td>${111} \langle 110 \rangle$ to ${111} \langle 112 \rangle$</td>
<td>${111} \parallel$ ND; prominent in bcc rolling textures</td>
</tr>
<tr>
<td>$\eta$</td>
<td>${001} \langle 100 \rangle$ to ${011} \langle 100 \rangle$</td>
<td>$&lt;100&gt; \parallel$ RD; near-relative of the $\beta$ fiber</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>${001} \langle 110 \rangle$ to ${111} \langle 112 \rangle$</td>
<td>$&lt;110&gt; \parallel$ TD</td>
</tr>
<tr>
<td>$\beta$</td>
<td>${112} \langle 110 \rangle$ to ${11118} \langle 4411 \rangle$</td>
<td>present in simulated bcc textures; prominent in fcc rolling textures</td>
</tr>
</tbody>
</table>
Fiber Plots bcc metals

- \( \gamma \) fiber only requires 30° along the full Range because of 3-fold \( <111> \) symmetry axis.
- \( \alpha \) fiber requires the full 90° Range
- Caution: Intensity Maximum \( NOT \) always at the ideal angular position.
Anisotropy and Symmetry

bcc Fibers in Euler space
**bcc fibers**: $\phi_2 = 45^\circ$ section

- $\gamma$, $<111>||ND$
- $\varepsilon$, $<110>||TD$
- Goss

**Fig. 6.** Exact positions of important orientations in the $\phi_2 = 45^\circ$ section.
Figure 5.10  Texture of cold rolled Fe-C steel with 0.45% C in the Euler space (75% rolling reduction). (a) Conventional representation of the ODF in $\varphi_1$-sections; (b) $\varphi_2 = 45^\circ$ section displaying the intensity distribution of the $\alpha$-fibre and $\gamma$-fibre orientations; (c) schematic representation of the most important orientations in bcc materials in the $\varphi_2 = 45^\circ$ section.
Anisotropy and Symmetry

(a) \( \alpha \)-fibre <011>\(/RD

(b) \( \gamma \)-fibre \{111\}/ND
**Other symmetry operators**

- Symmetry Operators of the *second kind*: these operators include Inversion center and Mirrors; determinant $= -1$.
- Inversion (= center of symmetry) simply reverses any vector: $(x, y, z) \rightarrow (-x, -y, -z)$.
- Mirrors operate through a mirror axis. An $x$-mirror is a mirror in plane $x=0$ $(x, y, z) \rightarrow (-x, y, z)$.
Summary

- Symmetry Operators in terms of Rotation matrices
  Construction from the axis-angle descriptions.
- Effect of symmetry on the range of Euler angles
- Particular effect of symmetry on certain named texture
  found in rolled fcc & bcc metals
- Definition of Various fibers